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ARITHMETICK
Compendiz'd ;
OR,
A short TREATISE
OF
Arithmetick.

Composed for the Use of the Charity Schools
erected by the Society in SCOTLAND
for propagating Christian Knowledge
Established by Her Royal Majesty Queen
ANNE Her Letters patent.

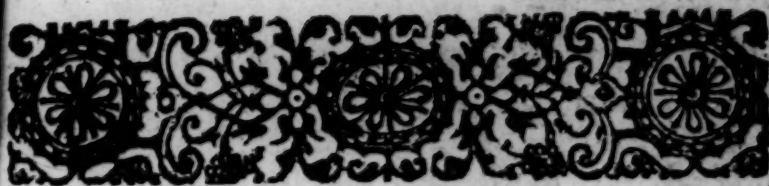
By a Member of the Society.

EDINBURGH,

Printed by the Heirs and Successors of *Andrew
Anderson*, Anno DOM. M.DCC.XIV.

John Graham
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If this book do hap
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and if an onast man had be
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TO THE
READER: .



*THE Title Page shews
the Design of Composing
this short Treatise ; and
nothing more is proposed
thereby, than so far to explain the common
Parts of Arithmetick, in whole
Numbers, and Fractions Vulgar and
Decimal, as may fit those for whom*

it is design'd for ordinary Business by a compleat Knowledge of what is necessary for them of Arithmetick. When any of them attain to such Knowledge and Circumstances as that their Affairs and Genius may lead them to look further into this useful Science, they may procure the Works of others, who have spent more Time and Pains than was thought needful for this. I do not pretend, and I believe no Body else, to write so fully and plainly on this Subject, as that the Beginner needs not the Help of a Teacher, but only to afford a small plain Piece at a cheap Rate to serve for common Use; And yet however small the present Performance is, it contains the first Grounds of Arithmetick, and so much as being well understood, will fit any for the Management of ordinary Affairs, either as Merchant, Factor,

Factor, Chamberlain, or Tradesman, and is a good Introduction to the Study of other more compleat Works of this Kind.

I have endeavoured to explain all the Rules with as much Brevity and Plainness as possible, and have industriously kept out several Things which swell other Books on this Subject, such as Multiplicity of Examples in each Rule, the Rules of Fellowship, Allegation, Barter, Loss and Gain, &c. all which are only so many Branches of that excellent Rule commonly called the Golden Rule, or Rule of Three. I have also forborn to insert many Tables to be found in other Books, and one Reason for my doing so is, That I never did see any trust to such Tables, because of the Uncertainty of their being right printed. I heartily wish, That all who undertake
Business

*Business wherein Arithmetick is need-
ful, did but thoroughly understand what
is herein, they would better acquit them-
selves, and prevent many Inconveniencies
and Losses that daily occur through
Persons undertaking Business which they
are not able to manage, through the Want
of Knowledge in the common Rules and
Practice of Arithmetick.*

*I have ordered the printing of this
small Book in such Manner, as it may
be bound up with Leaves of clean Paper
at the End of each Chapter, that the
Learner may thereon write down his own
Practice on each Rule.*

T H E

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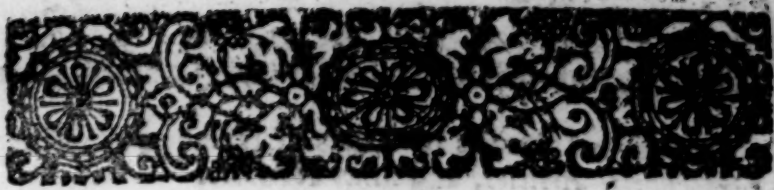
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ARITHMETICK



ARITHMETICK

Compendiz'd.

INTRODUCTION.



ARITHMETICK is the Art of Science, whereby we are taught rightly and dexterously to handle Numbers : And is ordinarily divided into three Kinds ; two of which are called *Vulgar*, and the third *Decimal Arithmetick*.

As to the Two *Vulgar Parts* ; The first is called *Arithmetick* in whole Numbers ; because every Character in it represents an entire Quantity of some Species or Thing propos'd. The second is called *Fractions*, which supposes an Unit, (or the Thing or Quantity represented) to be divided or broken into Parts.

As to the Third Kind, to wit, *Decimal Arithmetick* ; it is an artificial Invention for managing *Fractions* or *Broken Numbers* by an easy Way ; In which the several Operations differ little from those in whole Numbers.

B

The

The *Characters*, by which all Arithmetical Operations are perform'd, are these Ten, *Viz.*

1, 2, 3, 4, 5, 6, 7, 8, 9, 0.
One, two, three, four, five, six, seven, eight, nine, Cypher.

The several Species or Kinds of Numbers are Three, *viz. Digits, Articles, and Mixt Numbers.*

A *Digit* is any One of the first Nine Characters or Figures above singly expressed, as 1, 2, 3, &c.

An *Article* is any of the said Nine *Digits*, with a Cypher or Cyphers join'd to it on the right Hand, as

10,	30,	400, &c.
Ten,	Thirty,	Four hundred.

A *Mixt Number* is composed of several *Digits*; as

24,	17,	258, &c.
Twenty-four,	Seventeen,	Two hundred and fifty-eight.

Or it is composed of *Digits* and *Cyphers* intermixed, as,

201,	310,	3020, &c.
Two hundred and one,	Three hundred and ten,	Three thousand and twenty.

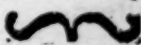
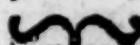
The principal Rules in Arithmetick are Five, *Viz. Numeration, Addition, Subtraction, Multiplication and Division.* Of which in Order.

CHAP. I.

Numeration.

NUMERATION teacheth, 1st. To read or express the Value of any Number when writ down. And 2^d. To write down any Number propos'd, according to its true Value.

In reading or writing down any Number, Two Things are to be observed : 1. The Place that any Digit possesseth. 2. The Value of each Digit in its Place. Both which are represented in the following Table.

Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
9	8	7.	6	5	4.	3	2	5.
								
Third Period.			Second Period.			First Period.		

By this Table it appears, That the Order of Places is reckoned from the right Hand towards the Left, each
 B 2 Place

Place exceeding the former Ten Times. But the Order of reading is from the left Hand to the Right, and that in each Period there are Three Figures.

The first Place is the Place of Units, so any Figure standing in it represents so many Units, as 5 is Five. The second Place is *Tens*, so any Figure standing in it, represents so many Tens, as 2 is Twenty. The third Place is *Hundreds*, so any Figure in it, represents so many Hundreds, as 3 is Three hundred, &c. And so the preceeding Number in the Table is read thus, viz. *Nine hundred eighty seven Million, Six hundred fifty four thousand, Three hundred and twenty five.*

5642. is read thus, Five thousand, six hundred, forty two.

60235. is read thus, Sixty thousand, two hundred, thirty five.

230280. is read thus, Two hundred thirty thousand, two hundred and eighty.

A ready Way in long Numbers, and for teaching Beginners to read and write them, is by pointing the Periods, thus,

Millions.	Thousands.	Hundreds.
5 6 4,	8 3 7,	2 9 1.

which is read thus, Five hundred sixty four Million, Eight hundred thirty seven thousand, Two hundred, Ninety one.

What is already taught on Nine Places in Numeration, being the fullest Extent needful for ordinary Practice, it is needless to enlarge any further on this first Rule or Part; which ought to be accurately taught, and clearly understood, before the Scholar proceed any further: It being often observed, That the not knowing Numeration perfectly, occasions Difficulty in learning and practising the other Rules.

But

But to fatisfie the curious Learner, here is one Example of more Places pointed as directed.

Millions of Millions.
Hundred thousands
of Millions.

Millions.

Thousands.

Hundreds.

6, 4 2 9, 3 5 7, 4 6 1, 2 8 2. which is read thus, Six Million of Millions, Four hundred twenty nine thousand three hundred fifty seven Million, Four hundred sixty one thousand, Two hundred eighty two.

In which observe, That there are Two Periods in the Place of Millions (for, as is already said, every Period has only Three Figures in it) and thus Numeration may be increased by repeating the Period of Millions as oft as needful, interjecting the Word Of so often as the Period is repeated.

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Book and if this Book
C H A P.

257..17~ 10~
 138 ~ 9 ~ 8
 25 ~ 4 ~ 6

257:17:10:
 138:12:2:
 37:9:8:
 25:4:6:
 459:2:
 25:6:4:
 257:17:10:

200
100
50
25

CHAP. II.

Addition of Integers.

ADDITION is the gathering together of several Sums, that so their *Total* may be known. In which observe these *Two Rules*; Rule 1. To set down each Number in its due Order, so as each Figure may be under those of the same Place, that is, *Units* under *Units*, *Tens* under *Tens*, *Hundreds* under *Hundreds*, &c.

Thus, suppose those Numbers 2567, 483, 97, and 8, were given to be added together: Set them down thus,

2567
483
97
8

Or thus, for its of
no Moment which
of the Sums be first
set down.

8
97
483
2567

3155 Total

Total 3155

Rule 2. Draw a Line at the Foot, under which set down the *Total*. Beginning at the Place of *Units*, add together all the Figures in that *Column* upwards, and if their *Sum* be under Ten, set it down below the Line in its proper Place: But if their *Sum* exceed Ten, you must set down only the *Excrecence* above Ten or Tens; and for each Ten in that *Column*, you must carry *One* to the next *Column*, which is *Tens*, adding what you carried, and all in the *Column* of *Tens* together, as you did in the *Units*, setting down the *Excrecence* above Ten or Tens, and so on to each Place and *Column* till all is completed. But if in any *Column* there is nothing above Ten or Tens, then place a *Cypher* under that *Column*, and carry the *Tens* to the next as before directed.

Take

Take this Example.

Begin at the Place of *Units*, and say 5. and 9 is 14, and 8 is 22, and 4 is 26; set down 6 under *Units*, and carry 2 to the Place of *Tens*; saying, 2 I carried and 8 is 10, and 2 is 12, and 3 is 15, and 5 is 20. There being nothing over the *Tens*, set down 0 and carry two to the Place of *Hundreds*; saying, 2 I carried and 3 is 5, and 4 is 9, and 7 is 16, and 6 is 22, set down 2, and carry two to the next Place; saying, 2 I carried and 9 is 11, and 8 is 19, set down 9 and carry One; which being added to 1 in the last Column, makes 2; which set down, and the Work is finished. So the *Total* of the Four Sums in the Example is, Twenty nine thousand two hundred and six.

18654
5738
429
385
29206

Examples for Practice,

78543	426	93572	4678
26725	321	4630	8245
42008	799	472	9554
70530	202	65340	256
14042	240	4785	375
21987	100	5246	78
65412	98	728	84
29078	72	282	987
10026	86	90	7
463	25	82	8
206	3	31	25
472	6	50	342
22	9	12	2183

C H A P. III.

*Addition of Sums of divers Denominations,
as of Money, Time, Weight, Measure, &c.*

Rule. HAVING placed down the Numbers given to be added, in their due Order, observing to set each Denomination in one Column, under other, *Viz.* Pounds under Pounds, Shillings under Shillings, Pence under Pence, &c. Consider how many Units of the least Denomination, make an Unit of the next superior Denomination, and how many Units you find of the higher Denomination, contain'd in the Column of the lower Denomination, so many must you carry to the Column of the higher Denomination, and if there be any over, set down such Overplus under its proper Column.

But this will be better understood by a Beginner, when he sees it in Example. Suppose then the following Sums of Money were given to be added together ; The Work will stand as below.

	<i>lib.</i>	<i>sh.</i>	<i>d.</i>
	257	17	10
	138	12	2
	37	9	8
	25	4	6
Total	459	4	2

In this Example, as in all of the like Nature, observe, That every 12 Pence make 1 Shilling, and every 20 sh. make 1 Pound, so here the Pence being added together, they amount to 26 ; which is twice 12 or 2 shill. and 2 over ; which 2 set down under the Line in the Place of Pence, and carry 2 to the shill. which being all added make 44 ; which is twice 20 or 2 lib. and 4 over ; which 4 set down in the Place of shill. and 2 carried, being joined with the Pounds, and the whole added, as is already taught in whole Numbers, the Total Sum is 459 4 : 2.

If the following Sums of *Weight* were given to be added, the Work will stand thus.

	<i>Stones</i>	<i>Pounds</i>	<i>Ounces</i>	<i>Drops</i>
In this Example it is to be observed, That	25	12	11	9
16 Drops make 1 Ounce,	9	8	10	12
16 Ounces make 1 Pound,	5	13	14	10
and 16 Pounds make 1 Stone.	4	5	6	14
So the whole Sum is	45	8	11	13
45 Stone, 8 Pound, 11 Ounce and 13 Drops.				

Examples for Practice.

<i>lib.</i>	<i>lb.</i>	<i>d.</i>	<i>Years.</i>	<i>Months.</i>	<i>Weeks.</i>	<i>Days</i>	<i>Stons.</i>	<i>Pounds.</i>	<i>Ounces</i>
35	17	11	25	4	2	5	1	12	13
76	19	10	2	10	3	6	2	4	1
8	10	2	7	8	1	4	3	11	7
33	6	8	5	2	2	3	3	5	11
40	13	4	2	9	1	2	2	6	4
56	9	8	8	6	3	4	1	3	0
66	13	4	5	7	2	6	2	0	4
89	9	6	2	3	1	4	4	14	9
33	5	8	4	11	2	5	7	3	15

Nota. When in Addition of divers Denominations the Sums in one Column (as of Pence or Ounces, &c.) amount to a great Total, so that you cannot easily distinguish how many you are to carry to the next Column, it will be fit that you put a Point at a certain Period, as at each 60 in Pence which makes 5 Shillings, or at each 80 in Ounces, which makes 5 Pounds, &c. which will make the Work very easie.

There is no certain Way to prove Addition, therefore its advised, That in all Operations thereof the Practitioner try his Work twice or thrice over, sometimes making the Addition upwards, and sometimes downwards; for in so doing he will not readily fall twice into one and the same Mistake.

CHAP. IV.

Subtraction in Integers.

BY Subtraction, One Number is deducted from another, that so the *Remainder* or *Difference* may be known. As to subtract 6 from 9, there will remain 3; This 3 is the *Difference* betwixt 9 and 6. Therefore in Subtraction the lesser Number is always to be set down below the greater, taking Care to place *Units* under *Units*, *Tens* under *Tens*, &c.

Rule. Having placed your Numbers as directed; Begin at the right Hand, or Place of Units, and subtract the lower Figure in that Place, from the Figure which stands over it, setting down the Remainder or Difference below the Line, in its proper Place, doing the like through all the Places.

If the Two Figures happen to be equal, set down a Cypher.

If the upper Figure be less than the lower, then add Ten to the upper Figure, or in your Mind call it Ten more, borrowing it from the next Place, and from that Sum subtract your lower Figure, setting down the Remainder as before directed. But because you added Ten to the former Place, (which was borrowed from the next superior Place) you must allow it back in that Place, by adding it to the lower Figure in that Place, when you come to subtract there. All which will appear in the following Example.

Borrowed by A. from B. _____	lib.
Paid by A. to B. _____	5842
	2751

Yet resting by A. _____	3091

The Operation is thus. Begin at the right Hand, and say, 1 from 2, there remains 1, which set down. Next 5 from 4, I cannot but from 14 (borrowing 10 as was directed to make it 14) there remains 9. Then proceed, saying, 1 I borrowed, and 7 is 8, from 8 remains nothing, for which place down a Cypher. Lastly, 2 from 3, remains 1, which set down, and the Work is finished.

Examples for Practice.

$$\begin{array}{r} 45926 \\ 19873 \\ \hline \end{array}$$

$$\begin{array}{r} 24865 \\ 4297 \\ \hline \end{array}$$

$$\begin{array}{r} 3570060 \\ 1902012 \\ \hline \end{array}$$

$$\begin{array}{r} 5347986000 \\ 1000490010 \\ \hline \end{array}$$

$$\begin{array}{r} 46002362 \\ 39420931 \\ \hline \end{array}$$

$$\begin{array}{r} 5347986000 \\ 1000490010 \\ \hline \end{array}$$

$$\begin{array}{r} 46002362 \\ 39420931 \\ \hline \end{array}$$

$$\begin{array}{r} 5347986000 \\ 1000490010 \\ \hline \end{array}$$

C H A P. V.

Subtraction in Numbers of different Denominations.

IN this I shall give the Examples only in Pounds, Shillings and Pence, which to any thinking Person will be sufficient to direct in all others of whatever Denominations.

In setting down the Numbers, take Care to place each Denomination in its proper Column, Pounds under Pounds, Shillings under Shillings, and Pence under Pence.

Thus done, begin at the Pence, then proceed to the Shillings, and lastly to the Pounds, as in the following Example.

$$\begin{array}{rcl}
 \text{Borrowed} & \text{---} & 235 : 14 : 8 \\
 \text{Paid} & \text{---} & 129 : 11 : 4 \\
 \hline
 \text{Yet resting} & \text{---} & 106 : 3 : 4
 \end{array}$$

In working say, 4 from 8, remains 4 Pence. Next, 11 from 14, remains 3 Shilling. So proceed to the Pounds, saying, 9 from 15, remains 6; then 1 I borrowed, and 2 is 3, from 3 remains 0; Lastly, 1 from 2 remains 1. Which compleats the Work, and the whole Remainder is 106 *lib.* 3 *sh.* 4 *d.*

But when it happens that the lower Number in the Pence or Shillings is greater than the Number above; you must, on such Occasion, borrow an Unit from the next superior Denomination, as 1 *sh.* which is 12 *d.* to supply the Place of Pence, and 1 *lib.* which is 20 *sh.* to supply the Place of Shillings, remembring always to allow what you borrowed when you come to the Place from whence you did borrow, as in the following Example.

$$\begin{array}{rcl}
 \text{Borrowed} & \text{---} & 152 : 12 : 8 \\
 \text{Paid} & \text{---} & 31 : 14 : 10 \\
 \hline
 \text{Yet resting} & \text{---} & 120 : 17 : 10
 \end{array}$$

In this Example say, 10 *d.* from 8 I cannot, but from 12 remains 2, which with the 8 makes 10 *d.* which set down,

down. Then go to the shill. saying, 1 I borrowed, and 14 is 15 from 12, I cannot, but from 20, remains 5, which with the 12 makes 17, which set down. Then proceed to Pounds, saying, 1 I borrowed and 1 is 2, from 2 remains 0, and so forward through all the Pounds as formerly directed, and the whole Remainder will be 120 lib. 17 sh. 10 d.

The following Examples are set down for the Learners further Instruction and Practice, whereof some are finished, the others to be wrought by himself.

	lib.	sh.	d.	lib.	sh.	d.
Borrowed	276	: 12	: 8	352	: 17	: 4
Paid	98	: 17	: 6	189	: 2	: 6

	Years	Months	Weeks	Days
From	25	: 08	: 2	: 5
Subtract	12	: 9	: 1	: 3
Remains	12	: 11	: 1	: 2

	Stones	Pounds	Ounces	Drops
Borrowed	12	: 10	: 2	: 0
Paid	8	: 9	: 14	: 12
	4	: 00	: 3	: 4

	Years	Months	Weeks	Days
From	4	: 5	: 1	: 2
Subtract	1	: 8	: 2	: 3

	Stones	Pounds	Ounces	Drops
Borrowed	2	: 4	: 5	: 12
Paid	1	: 12	: 11	: 14

The Proof of Subtraction is two Ways, 1. Add the Remainder to the lesser Sum, and that Total will be just the greater Sum. Or, 2. Subtract the Remainder from the greater Sum, and the Difference of those will be just the lesser Sum. As will appear thus.

Borrowed	12	: 17	: 8	Now add	4	: 11	: 4
Paid	8	: 6	: 4	To	8	: 6	: 4
Resting	4	: 11	: 4	makes	12	: 17	: 8

which is the greatest Sum.

Or, from	12	: 17	: 8
Subtract	4	: 11	: 4
Remains	8	: 6	: 4

which is the lesser Sum.

C H A P. VI.

Multiplication of Integers?

BY Multiplication, we increase or multiply one Number by another, so as to produce their whole Amount. One Number is said to be multiplied by another when a Product is found, containing the Number multiplied, so oft as there are Units in the Number multiplying.

In Multiplication there are three Numbers, or *Terms* to be observed.

1. The *Multiplicand*; or Number to be multiplied.

2. The *Multiplier*, or Number by which the Multiplication is made.

3. The *Product*, or Total proceeding from both. So if 15 be multiplied by 4, the Product will be 60.

The Operation is thus,

$$\begin{array}{r}
 \text{Multiplicand} \quad - \quad - \quad 15 \\
 \text{Multiplier} \quad - \quad - \quad - \quad 4 \\
 \hline
 \text{Product} \quad - \quad - \quad - \quad - \quad 60
 \end{array}$$

Thus you see the Product is 4 times the Multiplicand, as there are 4 Units in the Multiplier.

But before the Learner proceed in Multiplication, it is fit that he know the Product of any two *Digits* or single Figures when multiplied together, else he will be ready to err in his Operation, therefore he is to get the following *Table* exactly by Heart.

The Table is very plain, and needs no Illustration, only let the Learner observe, That in the several Steps or Periods of the Table, the Number of Places decreases by one, so that in the Period of 9, which is the last, there is but one Place, all other Multiplications of 9 being to be found in the preceeding Periods; For Example, 4 times 9 is 36, and so 9 times 4 is also 36, and so of all the rest.

Table

Table of Multiplication.

2 Times	{	2	is	4
		3	is	6
		4	is	8
		5	is	10
		6	is	12
		7	is	14
		8	is	16
		9	is	18
3 Times	{	3	is	9
		4	is	12
		5	is	15
		6	is	18
		7	is	21
		8	is	24
		9	is	27
4 Times	{	4	is	16
		5	is	20
		6	is	24
		7	is	28
		8	is	32
		9	is	36
5 Times	{	5	is	25
		6	is	30
		7	is	35
		8	is	40
		9	is	45
6 Times	{	6	is	36
		7	is	42
		8	is	48
		9	is	54
7 Times	{	7	is	49
		8	is	56
		9	is	63
8 Times	{	8	is	64
		9	is	72
9 Times		9	is	81

In Multiplication it is a Matter indifferent which of the two Numbers is made the Multiplicand, and which the Multiplier, for the Product will be the same, yet in Practice it is most convenient to make the Number that has fewest Places the Multiplier.

The Work of Multiplication is carried on from the right Hand to the left as in the former Rules. In which observe, at every particular Multiplication, to set down what is above Ten or Tens, and carry the
Tens

Tens to the next Place, and where there's no Excess, set down a Cypher.

Example,

$$\begin{array}{r} \text{Multiply} \quad - \quad - \quad - \quad 2563 \\ \text{by} \quad - \quad - \quad - \quad 4 \\ \hline \text{Product} \quad - \quad - \quad - \quad 10252 \end{array}$$

In this Example, say, 4 times 3 is 12, set down 2 and carry 1 to the next Place; saying, 4 times 6 is 24, and 1 I carried makes 25, set down 5 and carry 2. Then proceed, saying, 4 times 5 is 20, and 2 I carried is 22, set down 2 and carry 2. Then go forward, saying, 4 times 2 is 8, and 2 I carried is 10; which set wholly down because the Work is ended.

Thus, you see the whole Figures in the Multiplicand are multiplied by the Multiplier: And the like is done by every Figure of the Multiplier when there is more than one: Observing this *Rule*, to set down the first Product of each Figure under the Figure by which you are multiplying, carrying the rest forward to the left Hand, as you will see in the following Examples.

$$\begin{array}{r} \text{Multiply} \quad - \quad - \quad - \quad 3247 \\ \text{by} \quad - \quad - \quad - \quad 23 \\ \hline \end{array}$$

In this Example say, 3 times 7 is 21, set down 1 and carry 2; then 3 times 4 is 12, and 2 carried is 14, set down 4 and carry 1; then 3 times 2 is 6, and 1

$$\begin{array}{r} 9741 \\ 6494 \\ \hline \end{array}$$

$$\text{Product} \quad - \quad - \quad - \quad 74681$$

carried is 7, which set down; then 3 times 3 is 9, which set down; so the first Part is ended. Then begin with the second Figure of the Multiplier, saying, 2 times 7 is 14, set down 4 under the second Figure of the Multiplier, and carry 1; then 2 times 4 is 8, and 1 carried is 9, which set down; then 2 times 2 is 4, which set down; and lastly, 2 times 3 is 6, which set down; and the whole Multiplication is finished.

After the Multiplication is thus finished, add the several Products together, and the whole Product will be 74681 as in the Example.

D

Example

Example 3. Multiply - - - 85321
by - - - - 243

255963
341284
170642

Product 20733003

I shall not trouble to work this Example as the other two : But only desire the Learner to observe how the Products of each Figure in the Multiplier are advanced gradually, under the Figure by which the Multiplication is made.

The following Examples are set down for the Learner's Practice.

586732
2679

5280588
4107124
3520392
1173464

1571855028

576432
794

2305728
5187888
4035024
457687008

These and the other Examples, the Learner ought to practise on a separate Paper, and compare how his Operation agrees with the Example.

Nota. In many Cases the Work of Multiplication is abbreviate, viz.

1. When any Cyphers occur on the right Hand, either in the Multiplicand or Multiplier, or in both.

Rule. Multiply by the significant Figures (observing the Directions already given) and then annex all the Cyphers on the right Hand of the Product, as in the following Examples.

Multiply

Multiply - - 3600
by - - - 2300

108
72

Product - - 8280000.

Multiply - - 5742
by - - - 250

28710
11484

Product 1435500

2. When Cyphers occur in the Multiplier in the Middle thereof.

Rule. Place the Product of the Figure next to the Cyphers towards the left Hand, so many Places forward as there are Cyphers, supplying the Places of the Cyphers with Points, and not make whole Lines of Cyphers (as some have done) as in the following Examples.

Multiply - - 3942300
by - - - 720020

78846
78846
275961

Product 2838534846000

Multiply - - 345263
by - - - 402030

1035789
690526
1381052

Product 138806083890

3. When Cyphers occur in the Middle of the Multiplieand.

Rule. Set down the Figure which you was to carry from the former Multiplication, under the Place where the Cypher stands, and proceed to the next Figure, and

D 2

if

if you have nothing to carry, then supply that Place with a Cypher; as you will see in the Examples following.

$$\begin{array}{r} \text{Multiply} \quad - \quad 4602 \\ \text{by} \quad - \quad - \quad 39 \\ \hline \end{array}$$

$$\begin{array}{r} 41418 \\ 13806 \\ \hline \end{array}$$

$$\text{Product} \quad 179478$$

$$\begin{array}{r} \text{Multiply} \quad - \quad 450032 \\ \text{by} \quad - \quad - \quad 357 \\ \hline \end{array}$$

$$\begin{array}{r} 3150224 \\ 2250160 \\ \hline \end{array}$$

$$\text{Product} \quad 1350096$$

$$\text{Product} \quad 160661424$$

$$\begin{array}{r} \text{Multiply} \quad - \quad 40020 \\ \text{by} \quad - \quad 20030 \\ \hline \end{array}$$

$$12006$$

$$84..$$

$$\text{Product} \quad 9600600$$

4. When your Multiplier is 10, 100, 1000, &c. or any Article consisting of an Unit and Cyphers.

Rule. You have no more to do but add as many Cyphers to your Multiplicand as there are Cyphers in your Multiplier. Examples,

$$\begin{array}{ll} 254 \text{ Multiplied by } - - 10 \text{ is } - - & 2540. \\ 437 \text{ Multiplied by } - - 100 \text{ is } - - & 43700. \\ 726 \text{ Multiplied by } - - 1000 \text{ is } - - & 726000. \end{array}$$

5. When your Multiplier is any other Article, that is, any Digit above an Unit with Cyphers annexed, as 20, 300, 4000, &c.

Rule. Multiply the Multiplicand by the Digit, and add to the Product thereof, all the Cyphers annexed thereto, as in the following Examples.

$$\begin{array}{r} \text{Multiply} \quad - \quad 354 \\ \text{by} \quad - \quad - \quad 20 \\ \hline \end{array}$$

$$\text{Product} \quad - \quad 7080$$

$$\begin{array}{r} 42 \quad 212 \\ 300 \quad 4000 \\ \hline \end{array}$$

$$\text{Product} \quad 12600 \quad 848000$$

C H A P.

CHAP. VII.

Multiplication of Numbers consisting of several Denominations, the Multiplier being an Integer.

Rule. BEGIN at the least Denomination, multiplying by the Integer, and proceed gradually from the least to the greatest Denomination, carrying from the lower Denomination the Units belonging to the next greater Denomination, as in the following Example. Multiply 7 *lib.* 13 *sh.* 4 *d.* by 5. See the Work,

$$\begin{array}{r} 7 : 13 : 4 \\ 5 \\ \hline \text{Product for 5 Ells } 38 : 6 : 8 \end{array}$$

The Operation is thus, First 5 times 4 is 20 *d.* which is 1 *sh.* 8 *d.* set down the 8 *d.* and carry 1 to the *shill.* saying, 5 times 13 is 65 *sh.* and 1 carried makes 66 *sh.* which is 3 *lib.* and 6 *sh.* set down 6 *sh.* and carry 3 to the Pounds, saying, 5 times 7 is 35, and 3 carried makes 38, which set down, so you have the Product 38 *lib.* 6 *sh.* 8 *d.*

If your Multiplier consist of more Places than one, as 24, or 36, or 82, &c.

Rule. Multiply by such two Numbers as being multiplied together will make the Number of your Multiplier. Example,

At 4 *lib.* 6 *sh.* 8 *d.* the Ell, what will 24 Ells cost. See the Work.

Here the first Multiplication is by 4, which produces 17 : 6 : 8, and this Product multiplied by 6, the whole Product is 104 *lib.* as the full Answer to the

$$\begin{array}{r} 4 : 6 : 8 \\ 4 \\ \hline 17 : 6 : 8 \\ 6 \\ \hline 104 : 00 : 0 \end{array}$$

Question; for 4 times 6 is 24 : So the two Multiplications is equivalent to the multiplying by 24.

But

But in Case you cannot have two Numbers, which being multiplied together, will exactly make up the Multiplier; Then take the two Numbers that will come nearest to it, and work as in the last Example: And for what is wanting, multiply the Number given by the Number wanting, and add the Products of all the Multiplications together, and so you have the Answer.

Example.

At 12 *lib.* 6 *lb.* 8 *d.* the Boll, what will 78 Bolls cost. See the Work.

		12 : 6 : 8
1. Multiply by	- - -	8
Product		98 : 13 : 4
2. Multiply by	- - -	9
Product	- - -	888 : 00 : 0
Add	- - -	
Product of 12 : 6 : 8 multiplied by 6		74 : 00 : 0
Total Product	- - -	962 : 00 : 0

In the Example above, because there are not two Numbers which when multiplied together will make 78; I take the two that come nearest, viz. 8 and 9, which makes 72, and wants only 6. Then I multiply the Number given, first by 8, which makes 98 : 13 : 4, and this Product I multiply by 9, which makes 888 *lib.* To which I add the Product of the Number given, multiplied by 6, being 74 *lib.* which makes in all 962 *lib.* as the full Answer.

The true Knowledge of this Rule will direct how to multiply any Mixt Number, by any Number of Integers; By dividing your Multiplier into several Parts. As for Example. At 12 : 6 : 4 the Boll, what will 324 Bolls cost. See the Work.

Here

Here multiply first by 9,
the Product is 110 : 17 : 0,
which also multiplied by 9,
makes 997 : 13 : 0, which
being multiplied by 4 makes
3990 : 12 : 0 as the full

$$\begin{array}{r} 12 : 6 : 4 \\ \hline 110 : 17 : 0 \\ \hline 997 : 13 : 0 \\ \hline 3990 : 12 : 0 \end{array}$$

Answer. For the first Num-
ber being multiplied by 9,
and that Product again by 9, is equal to multiplying
by 81, because 9 times 9 is 81. And that Product
multiplied by 4 is equal to multiplying by 4 times 81,
or 324, for 4 times 81 is 324.

If there be any broken Number in the Question, as
suppose, at 3 lib. 5 sh. 6 d. the Boll, what will 9 Bolls
and a half cost.

Rule. Work first as directed in the foregoing
Examples for the Integer Bolls, and for the odd half
Boll, add the half of the Multiplicand.

$$\begin{array}{r} \text{See the Work} \quad - \quad - \quad - \quad \text{at} \quad 3 : 5 : 6 \\ \quad \quad \quad \quad \quad \quad \quad \quad \text{what} \quad \quad \quad \quad \quad \quad \quad \quad 9 \\ \quad \quad \quad \text{Product} \quad - \quad - \quad 29 : 9 : 6 \\ \text{To which add the half of } 3 : 5 : 6 \text{ is} \quad 1 : 12 : 9 \\ \quad \quad \quad \text{Total Answer} \quad - \quad - \quad 31 : 2 : 3 \end{array}$$

If the broken Number be three Quarters or one
Quarter, &c. you are to add proportionally for the
same.

Example, at 7 : 6 : 8 the Boll, what will 8 Bolls
and 3 Firlots cost. Nota. The 3 Firlots is 3 Quarters
of a Boll.

$$\begin{array}{r} \text{See the Work} \quad - \quad - \quad - \quad \text{at} \quad 7 : 6 : 8 \\ \quad \quad \quad \quad \quad \quad \quad \quad \text{what} \quad - \quad - \quad \quad \quad \quad \quad \quad \quad \quad 8 \\ \quad \quad \quad \text{Product} \quad - \quad - \quad 58 : 13 : 4 \\ \text{To which add 3 Quarters of } 7 : 6 : 8 \text{ is} \quad 5 : 10 : 0 \\ \quad \quad \quad \text{Total Answer} \quad - \quad - \quad 64 : 3 : 4 \end{array}$$

The

Or thus,

at	-	-	-	7 : 6 : 8
			what	8

				58 : 13 : 4
Half a Boll				3 : 13 : 4
A Firloz	-	-		1 : 16 : 8

Total				64 : . 3 : 4
-------	--	--	--	--------------

The only sure Proof of Multiplication is by Division; Therefore I shall forbear treating upon it, until I have also taught the Rule of Division; and indeed it will then most naturally fall in: Because, as Multiplication is proven by Division; so Division is proven by Multiplication: Any other Way being fallacious and uncertain, such as throwing out the Nines.

• C H A P.

William

C H A P. VIII.

Division in Integers.

DIVISION teacheth, to discover, how oft one Number is contain'd in another.

In this Rule, there are *Three Numbers* or Parts to be noticed, viz.

1. The *Dividend*, or Number to be divided.
2. The *Divisor*, or Number by which the Division is made.
3. The *Quotient* or Answer; which shews how oft the Divisor is contain'd in the Dividend.

Sometimes also there is a 4th Number, which arises, when the Dividend cannot be fully divided by the Divisor into equal Parts; which is called *the Remainder*, and is always less than the Divisor; as will afterwards appear.

In Division, it's to be observed, That as oft as an Unit is contain'd in the Divisor, so oft does the Dividend contain the *Quotient* or Answer.

Division is either *single* or *compound*.

Single Division is, when the Divisor is one Figure or Digit, as 2, 4, 5, 7, 8, &c.

Compound Division is, when the Divisor consists of Two, Three or more Figures, as 23, 352, 2685, &c.

Nota. In all Operations of Division, the Course of the Work is from the left Hand to the Right; which is contrary to the preceeding Rules, which are wrought from the right to the left Hand.

I shall begin with *single Division*. Suppose then 628 Pounds given to be divided amongst 4 Men, what will each Man's Part be.

Rule. Place your Dividend betwixt two crooked Lines, thus) 628 (and set your Divisor in the Line before the Dividend, leaving the other Line for your Quotient, and then proceed as follows.

E

Dividend

Example 1.

Dividend Quotient or
Divisor 4) 628 (157 Answer

First put a Point under 6 the first Figure of the Dividend, and say, how oft is 4 in 6 (Answer) 1, which Place in the Quotient, saying once 4 is 4, which being set under and subtracted from 6, remains 2; then Point and bring down the next Figure 2, which will make 22, saying, 4 in 22 is 5 times, place 5 in your Quotient, saying, 5 times 4 is 20, which set under and subtracted from 22, remains 2; then point and bring down 8 making 28, say 4 in 28 is 7 times, place 7 in the Quotient, saying, 7 times 4 is 28, which placed under and subtracted from 28, there remains nothing; so the Work is finished; and the Answer to the Question stands in the Quotient 157: Which is as much as to say, that 628 Pounds divided amongst 4 Men, each one will get 157 Pounds.

If the first Figure of your Dividend be not greater, or at least equal to your Divisor, then you must take in another Figure, and put your first Point below it, for your first Dividend; as in this 2d Example,

In 12096 Days, how many Weeks. Here 12096 is to be divided by 7, because there are 7 Days in one Week. See the Work.

7) 1204 (172 Weeks.

$$\begin{array}{r}
 7 \overline{) 1204} \\
 \underline{7} \\
 50 \\
 \underline{49} \\
 14 \\
 \underline{14} \\
 0
 \end{array}$$

In

In the preceeding Example, because 7 the Divisor cannot be had in 1, the first Figure of the Dividend, therefore the first Point is put under 2, the second Figure of the Dividend. So you proceed thus, saying, 7 in 12 is 1, place 1 in the Quotient, saying, 1 time 7 is 7, which put under and subtracted from 12, remains 5; then point and bring down 0 making 50, say 7 in 50 is 7 times, place 7 in the Quotient, saying, 7 times 7 is 49, which set under and subtracted from 50, there remains 1. Lastly, Point and bring down 4, making 14, say 7 in 14 is 2 times, set 2 in the Quotient, saying, 2 times 7 is 14, which set under and subtracted from 14, remains nothing. So the Work is ended. And the Answer is, that in 1204 Days are 172 Weeks.

Nota. The pointing of the Figures in the Dividend, is of two Uses. 1. To show how far you have proceeded in your Work. 2. To show how many Figures ought to be in the Quotient. For as many Points as there are in the Dividend, so many Figures must there be in the Quotient; As is evident in the Examples.

If at any Time, in the Middle of an Operation, it happen that the former Remainder and Figure pointed and brought down is less than the Divisor, you must place a Cypher in the Quotient, and point and bring down another Figure, and so oft as thus there is a Defect, as oft must you place a Cypher in the Quotient; And if there be such a Defect after bringing down the last Figure of your Dividend, then place a Cypher in your Quotient, and the Operation is ended.

Example 3. Suppose 1050700 given to be divided by 35.

See the Work.

$$35 \overline{) 1050700} \quad (30020$$

$$\begin{array}{r} 105 \\ \hline 0070 \\ 70 \\ \hline 00 \end{array}$$

In this Example, because 35 cannot be found in the first two Figures of the Dividend, therefore the first Point must be under the

E 2

third

third Figure ; and the first Operation is, how oft 35 is to be found in 105. But this, and the like, where there are many Figures, being uneasy and impracticable for the Memory ; Therefore, the best Way is, to try how oft the first Figure of the Divisor, is contain'd in the first of the Dividend, and then to proceed ; With this Caution, That in Case the second Part or Figure of the Divisor, cannot also be found as oft in the second Figure of the Dividend, when join'd to the Remainder of the first Figure (if any be) then you are to take your quotient Figure one less than before. And it will often occur, that tho' you can find the first and second Figures of your Divisor and Dividend, to answer to your quotient Figure, which you first found, yet you will find the third, and sometimes the fourth to be defective, therefore, according to the greatness of your Divisor, you must, by your Memory, try the Figure of your Quotient, through the first, second and third, and sometimes the fourth Figures of your Dividend, before you proceed to Work ; but generally a Trial in three Figures will be sufficient : Which will partly appear in the next Example.

Now as to the preceeding Example the Operation is thus. First say how oft 3 in 10, is 3 times, set 3 in the Quotient, by which multiplying the Divisor 35, setting the Product below, and subtracting it from 105, the first Point of your Dividend, there remains nothing or 0, to which point and bring down 0, and because the Divisor cannot be found therein, therefore put 0 in the Quotient ; then point and bring down 7, and because the Divisor cannot be found therein, therefore put another 0 in the Quotient ; then point and bring down 0, then ask how oft 3 in 7, is 2 times, place 2 in the Quotient, by which multiplying your Divisor and setting the Product below, and subtracting it from 70, there remains 0 ; lastly, point and bring 0, and because your Divisor is not to be found therein, put 0 in the Quotient, and the Work is ended. And the Answer is 30020.

Example

Example 4. Let it be required to divide 123156 by 36.

See the Work.

$$36 \overline{) 123156} \quad (3421$$

In this Example altho' 3 the first Figure of the Divisor, be 4 times in the first Part of the Dividend, yet you will not find 4 times 6 in 3, therefore you must take 3 for your quotient Figure, by which having multiplied 36, and set the Product under, and subtracted it from 123, the first Point of the Dividend, there remains 15; then point and bring down 1, makes 151; Then try how oft 3 is in 15, and you will find it can be only 4 times, set 4 in the Quotient, then multiply the Divisor and subtract as formerly, there will remain 7; then point and bring down 5, makes 75; ask how oft 3 in 7 you will find it 2 times, set 2 in the Quotient, by which multiply the Divisor, and set the Product under and subtract it from 75, there remains 3; Lastly point and bring down 6, makes 36, which being equal to the Divisor set 1 in the Quotient, and nothing remains, so the Work is ended, and the Answer is 3421.

$$\begin{array}{r} 108 \\ \hline 151 \\ \hline 144 \\ \hline 75 \\ 72 \\ \hline 36 \\ 36 \\ \hline 0 \end{array}$$

Example 5th. By three Figures.

Let it be required to divide 379302 by 231.

See the Work, 231) 379302 (1642

The Manner of working this Example being the same as in the former, I shall forbear to explain the same.

$$\begin{array}{r} 231 \\ \hline 1483 \\ 1386 \\ \hline 970 \\ 924 \\ \hline 462 \\ 462 \\ \hline 0 \end{array}$$

Let

Let the Learner study the Knowledge of the preceeding Examples. By 1, 2 and 3 Figures in the Divisor, and he will understand how to work by any Number of Figures.

If any *Remainder* happen to be after your Division is finished ; that *Remainder* is the *Numerator* of a *Fraction*, and the Divisor is the *Denominator*, the Value whereof is to be discovered, as in the following Example.

Example 6th.

Let it be required to divide 368424 Pounds amongst 1728 Men, that each Ones Share may be known in Pounds, Shillings, and Pence.

See the Work. lib. sh. d.

1728) 368424 (213 : 4 : 2

3456

2282

1728

5544

5184

360 Remainder is ³⁶⁰/₁₇₂₈

Shill. in a Pound

20

1728) 7200 (4 Shill.

6912

288 Remainder.

Pence in a Shilling - - 12

576

288

1728) 3456 (2 Pence.

3456

0

After

After you have finished your Division and found the Quotient of 213 *lib.* There is a Remainder of 360 Pounds, which is $\frac{360}{12}$ Parts of a Pound, for finding the Value whereof in Snillings and Pence, first multiply this Remainder 360 by 20, produces 7200 Shillings, which divided by the first Divisor, 1728, you will find 4 Shillings, which place in your Quotient in the Place of Shill. and there is a Remainder of 288 Shill. which multiplied by 12, produces 3456 Pence, which being divided by your first Divisor, you will find 2 Pence and nothing remaining, set down 2 in your Quotient in the Place of Pence, and the Work is ended, and the Answer is 213 *lib.* 4 *sb.* 2 *d.* to each Man.

If there had been any Remainder in the Pence, it would have been the Fraction of a Penny.

The Nature of a Fraction will be explain'd when we come to treat of Fractions.

Because the Work of Division proves uneasy to Beginners, for when the Divisor consists of many Places, he is often uncertain what Figure to put in the Quotient. As a Help in this, let him make a Table of the Divisor multiplied by the Nine Digits, in which he will find the quotient Figure and its Multiplication for every Dividual.

Example.

Let it be required to divide 1461162 by 234.

Table of the Divisor by
the Nine Digits.

1	-	-	-	234
2	-	-	-	468
3	-	-	-	702
4	-	-	-	936
5	-	-	-	1170
6	-	-	-	1404
7	-	-	-	1638
8	-	-	-	1872
9	-	-	-	2106

234) 1460862 (6243

1404

568

468

1006

936

702

702

In the Example above, the first Dividual is 1460, the Number in the Table nearest to it and less is 1404, which stands against 6; place 6 in the Quotient, and set down 1404 under the Dividual, and subtract as formerly directed, so there remains 56; to which point and bring down 8, making 568 for the next Dividual; the Number in the Table nearest to it is 468, and stands against 2; Place 2 in the Quotient, and set down 468, and subtract as formerly, there remains 100; to which point and bring down 6, making 1006, for the next Dividual: The Number nearest it is 936, standing against 4; place 4 in the Quotient, and set down 936, and subtract as before, there remains 70; to which point and bring down 2, making 702 for the last Dividual; the Number answering to it is just 702 and stands against 3, place 3 in the Quotient, and set down 702. So the Work is ended, and there is no Remainder.

Nota. In consulting your Table, be sure never to take a Number higher than your Dividual.

Contractions in Division.

1. **I**F your Divisor be one of the Nine Digits singly, as 2, 3, 4, 5, &c.

Rule. You need only take the Half, Third, Fourth, Fifth, &c. Part of the Number given, as in the following Examples.

Divide 7642 by 2.
One Half is $\frac{7642}{3821}$ Quotient.

Which is performed thus; beginning at the left Hand, say, the Half of 7 is 3, which set down, and 1 remaining join'd to 6 makes 16, the Half whereof is 8, which set down; next the Half of 4 is 2, which set down; lastly,

lastly, the Half of 2 is 1, which set down; so your Quotient or Answer is 3821.

Divide 7426 by 3.

One Third is 2475 $\frac{1}{3}$ Quotient

Perform'd thus, One third of 7 is 2, One third of 14 is 4, One third of 22 is 7, and one third of 16 is 5, and there remains 1 which is $\frac{1}{3}$.

Divide 67453 by 5.

13490 $\frac{3}{5}$ Quotient

Perform'd thus, One fifth of 6 is 1, One fifth of 17 is 3, One fifth of 24 is 4, One fifth of 45 is 9; and lastly, One fifth of 3 is 0; so there remains 3, which is $\frac{3}{5}$.

These three Examples duly considered, are sufficient to direct in all others.

2. If your Divisor be an Unit, having Cyphers on the right Hand.

Rule. Cut off from the right Hand in your Dividend so many Figures as there are Cyphers in your Divisor, and the Figures cut off on the right Hand are the Remainder.

Suppose then 46575 Pounds were given to be divided amongst 10, or 100, or 1000 Persons, the Answers will be as follows.

First. If among Ten, the Answer is 4657 *lib.* 10 *sh.* to each, thus - - - 46|575. The 5 cut off is $\frac{5}{10}$ or 10 *sh.*

Secondly. If among 100, the Answer is 465 *lib.* 15 *sh.* to each, thus - - - 4657|5. The 75 cut off is $\frac{75}{100}$ or 15 *sh.*

Thirdly. If among 1000, the Answer is 46 *lib.* 11 *sh.* 6 *d.* to each, thus - - - 46|575. The 575 cut off is $\frac{575}{1000}$ or 11 *sh.* 6 *d.*

The Value of the above Remainders or Fractions is found by the Rule in the 6th Example.

3. If your Divisor and Dividend have each of them Cyphers on the right Hand.

F

Rule.

Rule. Cut off an equal Number of Cyphers in both, and proceed to Work with the remaining Figures.

So if 7652000, were to be divided by 26000. Having cut off three Cyphers in each, there will remain 7652 to be divided by 26.

4. If your Divisor have Cyphers on the right Hand, and that there is none in the Dividend.

Rule. Cut off so many Figures in the Dividend, as there are Cyphers in the Divisor, cutting off also the Cyphers from the Divisor, and proceed in your Work with the Remainders, with this Caution, That you add the Figures cut off in the Dividend, to the Remainder after your Work is ended, which will be a Numerator to your whole Divisor, as in the following Example.

Suppose 367842 Pounds given to be divided amongst 2200 Persons.

See the Work.

$$22 \overline{) 367842} \quad (167 \frac{442}{2200}$$

22

147

132

158

154

Remains - - 4 to which join 42 cut off, makes 442 as the Numerator to the Divisor 2200; so the Quotient as above is 167 $\frac{442}{2200}$, the Value of which Fraction you may find, by the Directions in the 6 Example.

5. If your Divisor be any of the Digits above an Unit, with Cyphers annexed, as 20, 200, 30, 300, 4000, 50000, &c.

Rule.

Arithmetick compendiz'd.

35

Rule. Cut off so many Figures from the right Hand of your Dividend, as there are Cyphers cut off in your Divisor, and divide the Remainder by the Digit, according to the Directions in the first Rule of Contractions.

Example. Divide - - 2485000 by 200.

The Work is thus $2 \overline{) 00) 24850 \overline{) 00}}$

One Half is 12425 the Answer.

Divide 53426 by 400, the Work will be thus - - $4 \overline{) 00) 534 \overline{) 26}}$

One Fourth is - 133 $\frac{226}{400}$ the Answer.

Divide 8976432 by 6000, the Work will be thus - - - $6 \overline{) 000) 8976 \overline{) 432}}$

One Sixth is - - - 1496 $\frac{432}{6000}$ the Answer.

In these Examples, observe, That the Remainders are set as Numerators to the whole Divisor, the Value whereof is discovered by the Directions in the 6th Example.

Proof of Multiplication and Division.

AT the End of Multiplication I advertised the Learner, That Multiplication and Division did mutually prove each other.

1. For Multiplication. Divide your Product by your Multiplier, the Quotient will be your Multiplicand exactly.

2. For Division. Multiply your Quotient by your Divisor, the Product will be your Dividend exactly.

Example for both.

Multiply - - - 41725
by - - - - 632

83450

125175

250350

632) 26370200

2528

1090

632

4582

4424

1580

1264

3160

3160

0

Proof of Multi-
plication.

(41725

632

83450

125175

250350

26370200

Proof of
Division.

In the Example above, the Product of the Multiplication which is 26370200 being divided by the Multiplier 632, the Quotient produces 41725 which is the Multiplicand, which proves the Work of Multiplication.

Next. The Quotient 41725 being multiplied by 632 the Divisor, the Product is the same with the Dividend, to wit 26370200; which proves the Work of Division.

C H A P.

CHAP. IX.

Reduction.

REDUCTION is the Rule whereby Sums are changed from one Denomination to another, whether higher or lower. As Pounds into Shillings or Pence : Or Shillings or Pence into Pounds.

So Reduction is of two Kinds, *Ascending* or *Descending*.

Reduction ascending is, when we are to reduce a Number of a lesser Denomination to a Number of a higher Denomination, as Pence or Shillings into Pounds ; which is done by Division.

Rule. Divide the Number given, by such a Number of Units of the Denomination given, as make an Unit of the superior Denomination, and the Quotient is the Answer.

Example 1. Reduce 1728 Pence into Shillings.

See the Work.

12) 1728 (144 Shillings

12

—
52
48
—
48
48
—
0

Here the Pence given, are divided by 12 because 12 d. make 1 *sh.* and the Quotient is 144 ; which shows that 1728 d. is equal to 144 *sh.*

If the Reduction be two (or more Steps) as Reduction of Pence into Pounds.

Rule. Make your Operation gradually to each Denomination, till you come to the highest.

Example

Example 2. Reduce 29280 Pence into Pounds.

See the Work.

12) 29280 (2440 Shill.

Here the first Division is by 12, which produces 2440 Shill. which being divided by 20 (because 20 Shill. make 1 Pound) produces 122 Pounds for the Answer.

$$\begin{array}{r}
 24 \\
 \hline
 52 \quad 2 \overline{) 2440} \\
 48 \quad \frac{1}{2} \text{ is } 122 \text{ Pounds} \\
 \hline
 48 \\
 \hline
 48 \\
 \hline
 00
 \end{array}$$

If in an Operation of this Kind, there happen a Remainder in any of the Divisions, such Remainder is of the same Denomination with the Sum divided.

Example 3. In 1996 d. how many Pounds, Shillings and Pence.

See the Work.

12) 1996 (166 Shillings

Here the several Divisions being made, the Answer is 8 lib. 6 sh. 4 d.

$$\begin{array}{r}
 12 \quad \frac{1}{2} - 8 \text{ Pounds} \\
 \hline
 79 \\
 72 \\
 \hline
 76 \\
 72 \\
 \hline
 4
 \end{array}$$

Remains - - - 4 d.

Reduction descending is, when we bring a Sum of a greater Denomination, into a Sum of a lesser Denomination: As Pounds into Shillings or Pence, &c. which is performed by Multiplication.

Rule. Multiply the Number given, by such a Number of Units of the inferior Denomination, as make an Unit of the higher Denomination, and the Product is your Answer.

Example

Arithmetick compendiz'd.

Example 1. In 224 Pounds, how many Shilling

See the Work.

Here you multiply the Number given by 20 (because 20 *sh.* make 1 Pound) and the Product is 4480 Shillings for your Answer.

$$\begin{array}{r} 224 \\ 20 \\ \hline 4480 \end{array}$$

When the Reduction has two (or more) Steps, as reducing Pounds to Pence.

Rule. Make your Operation gradually to each Step till you come to the lowest:

Example 2. In 1352 Pounds, how many Pence.

See the Work.

Here, you first reduce the Pounds to Shillings by multiplying by 20, so you have 27040 Shill. Then you reduce those Shillings to Pence by multiplying by 12, so the last Product is 324480. Which is the Number of Pence in 1352 Pounds.

$$\begin{array}{r} 1352 \\ 20 \\ \hline 27040 \text{ Shillings} \\ 12 \\ \hline 5408 \\ 2704 \\ \hline 324480 \text{ Pence} \end{array}$$

These Examples and Directions, when duly considered, are sufficient to illustrate the Rule of Reduction. But for the Learner's Instruction I shall subjoin a few Examples which the Learner may work himself.

In 245 Days how many Hours? Answer 5880. Found by multiplying by 24, which is the Number of Hours in one Day.

In

In 350 Bolls how many Firlots? Answer 1440.
Found by multiplying by 4, which is the Number of Firlots in one Boll.

In 23 Years, how many Months, Weeks, or Days?
Answer 276 Months, or 1104 Weeks, or 7728 Days,
Found by multiplying first by 12 for Months, then that Product by 4 for Weeks, and the last Product by 7 for Days.

In 5880 Inches how many Ells? Answer 163 Ells and 12 Inches. Found by dividing by 36, which is the Number of Inches in one Ell.

In 19230 Pence, how many Shillings or Pounds?
Answer, 1602 Shillings and 6 Pence, or 80 *lib.* 2 *sh.* 6 *d.*
Found by Division.

C H A P.

CHAP. X.

The Rule of Three, or Golden Rule.

THIS Rule is called the *Golden Rule*, because of its extraordinary Usefulness, in all Arithmetical Questions and Operations.

It is also called the *Rule of Proportion*, because, from certain Numbers given and known, another unknown and sought, is found out.

This Rule is either *Single* or *Compound*.

The *single Rule of Three* is, when three Numbers are given, and a fourth proportional to them is sought.

The *Compound Rule of Three* is, when five Numbers are given, and a sixth proportional to them is sought.

As to the *single Rule*, Observe, first, That some Questions are resolved by multiplying the 2d Number by the 3d, and dividing that Product by the first Number, and the Quotient is the Answer, which is called the *Direct Rule*. Other Questions are resolved by multiplying the second Number by the first, and dividing the Product by the third Number, which is called the *Indirect Rule*.

The first Thing to be noticed, is the right stating of the Numbers given; for doing whereof, observe this

Rule. Suppose this to be the Question, If 4 Students spend 19 Pounds, what will 8 Students spend? The Number upon which the Question is raised, must stand in the third Place, and the other which is of the same Denomination, must be in the first, and consequently, the other Number of a different Denomination, must possess the Middle Place.

Stud. lib. Stud.

As follows, if - - 4 : 19 :: 8

The first and third Numbers are called the Extreams, and the Middle Number the Mean.

In which Question, a fourth Number is sought, which must bear such Proportion to the second, as the third doth to the first. That is, as 8 is double 4, so the Number sought must be double 19. For if 4 Students spend 19 Pounds, 8 Students will spend twice 19 Pounds.

See the Work.

Here according to the Directions given, 19 the Middle Number, being multiplied by 8 the third Number, produces 152, which being divided by 4 the first Number, the Quotient is 38, which is the Answer; and so is set down as the 4th Number required.

ft.	lib.	ft.	lib.
4	: 19 ::	8	: 38.
		8	Pounds.
4)	152	(	38 Answer.
	12		which compleats
	32		the Proposition,
	32		and answers the
	0		Question.

Example. Of the Indirect Rule.

If a certain Quantity of Hay maintain 8 Horfe for 12 Days, how long will the same Quantity maintain 16 Horfe.

Here, as 8 is to 16, so is 12 to the Number sought. And therefore as 8 is the Half of 16, so the Number sought will be the Half of 12.

See the Work.

8	:	12	::	16	:	6.
		8	Days			
16)	96	(	6 Answer.			
	96					
	0					

In this Example the 2d Number multiplied by the first, produces 96, which being divided by the third Number, the Quotient is 6 for the Answer of the Question, and is set down in

ccc ou

in the fourth Place : So if 8 Horses be maintain'd 12 Days by a certain Quantity of Hay ; the same Quantity will maintain 16 Horses only 6 Days.

The two preceeding Questions being very plain, and their Resolution very natural, even without Skill in Arithmetick ; The Rules are thereby clearly illustrate.

But because there is great Variety of Questions that are resolv'd by the Rule of Three, yea infinite Variety ; and that all Learners are difficulted to discover, whether the Operation be by the direct or indirect Rule, to make this easy, observe this,

Rule. Consider whether the Number you are seeking, be greater or less than the Middle Number, (which will be obvious to a very mean Capacity) and if it be greater, then the lesser Extream is the Divisor. If it be less than the Middle Number, then the greater Extream is the Divisor. And having found which of the two Extreams is the Divisor, it will appear whether the Operation is by the direct or indirect Rule.

Example of the direct Rule.

If 13 Packs of Cloth cost 326 Pounds, what will 39 Packs cost.

See the Work.

$$13 : 326 :: 39 : 978.$$

$$\begin{array}{r} 39 \\ \hline 2934 \\ 978 \end{array}$$

Pounds.

$$13) 12714 \quad (978 \text{ Answer,}$$

$$\begin{array}{r} 117 \\ \hline 101 \\ 91 \\ \hline 104 \\ 104 \\ \hline 0 \end{array}$$

Which is set down as the fourth Number.

Example. Of the Indirect Rule.

If 55 Mafons build a Houfe in 15 Months, how
foon would 165 Mafons build it.

See the Work.

$$55 : 15 :: 165 : 5$$

55

75

75

Months

165) 825 (5 Answer.

825

0

In Cafe any of the Numbers given be of divers Denominations, you muſt reduce the ſame to the loweſt Denomination. As Pounds, Shillings and Pence muſt be reduced to Pence, or Years, Months and Weeks muſt be reduced to Weeks, &c.

Obſerving. That if any one of the two Extreame be reduced, the other Extreame muſt alſo be reduced alike with it.

And if the Middle Number be reduced, you are to notice, That the Answer will be in the ſame Denomination, to which your Middle Number is reduced. All which will appear in the Examples following.

If 64 Ells of Stuff coſt 38 lib. 8 ſh. what will 5 Ells coſt.

See the Work.

lib. ſh.

38 : 8

20

makes 768 Shill.

$$64 : 768 :: 5 : 60$$

5

64) 3840 (60 Shill,

384

00

In the Queſtion above, the Middle Number is reduced to Shill. ſo the Answer is 60 Shill.

If

Arithmetick compendix'd.

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If 6 Ells. of Stuff cost 3 lib. 12 sh. 6 d. what will 64 Ells and a Quarter cost.

Before you state your Question, you must reduce your first and third Numbers to Quarters; and your Middle Number to Pence; which is thus,

$$\begin{array}{r}
 6 \\
 4 \\
 \hline
 24 \text{ Quarters}
 \end{array}
 \qquad
 \begin{array}{r}
 3 : 12 : 6 \\
 20 \\
 \hline
 72 \\
 12 \\
 \hline
 150 \\
 72 \\
 \hline
 870 \text{ Pence.}
 \end{array}
 \qquad
 \begin{array}{r}
 64 : \\
 4 \\
 \hline
 257 \text{ Quarters.}
 \end{array}$$

Then your Question will stand as follows.

$$\begin{array}{r}
 \text{Quart.} \quad \text{Pence.} \quad \text{Quart.} \\
 24 : 870 :: 257 : 9316 \text{ Pence.}
 \end{array}$$

$$\begin{array}{r}
 609 \\
 435 \\
 174 \\
 \hline
 24) 223590 \quad (9316 \text{ Answer, which being} \\
 216 \quad \text{reduced according} \\
 75 \quad \text{to the Rule of Re-} \\
 72 \quad \text{duction, amounts to} \\
 39 \quad 38 : 16 : 4. \\
 24 \\
 150 \\
 144 \\
 \hline
 6
 \end{array}$$

If 52 Ells 3 Quarters of 6 Quarter broad Stuff hing a Room, how many Ells of Ell-broad Stuff will hing the same Room.

Here

Here the Question arises upon the Breadth of the Stuff.

See the Operation.

$$6 : 52\frac{1}{2} :: 4 : 79\frac{1}{2} \text{ Ells.}$$

$$\begin{array}{r} 4 \\ 211 \text{ Quarters.} \\ 6 \end{array}$$

4) 1266 (316 : Quarters, which reduced to Ells, makes 79 Ells and $\frac{1}{2}$ as the Answer.

$$\begin{array}{r} 12 \\ \underline{12} \\ 6 \\ \underline{6} \\ 4 \\ \underline{4} \\ 26 \\ \underline{24} \\ 2 \end{array}$$

① If a Servant's weekly Pay be 3 *lib.* 4 *sh.* and is forborn for 2 Years 6 Months and 3 Weeks, what will then be due to him.

See the Work.

$$\begin{array}{r} 3 \text{ lib. } 4 \text{ sh.} \quad 2 \text{ Y. } 6 \text{ Mo. } 3 \text{ Weeks} \\ 20 \quad 12 \\ 64 \text{ Shill.} \quad 30 \text{ Months. Then state the Quest. thus} \\ 4 \quad 123 \text{ Weeks} \end{array}$$

W. sh. W.

$$1 : 64 : 123$$

$$\begin{array}{r} 64 \\ 492 \\ 728 \end{array}$$

Here the weekly Pay is reduced to Shillings, so the Answer is in Shillings; which being again reduced makes 393 *lib.* 12 *sh.* and the Years, Months and Weeks are all reduced to Weeks. As in the Operations above.

Answer $\frac{787}{2}$ Shill.
which is 393 *lib.* 12 *sh.*

If for 4 Weeks or 28 Days Diet, I pay 14 *lib.* 13 *sh.* 4 *d.* what must I pay for a Year, which is 365 Days.

See

Arithmetick compendiz'd.

See the Work.

lib. lb. d. lib. lb. d.
28 : 14 : 13 : 4 :: 365 : 191 : 3 : 9 ²¹/₂

20
293
12
590
293
3520 Pence
365
1760
2112
1056

28) 1284000 (45885 ²¹/₂ Pence. Which being reduced,
..... makes for the Answer 191 lib. 3 lb.
112 9 ²⁰/₂₁ d.

112
164
140
248
224
240
224
160
140

Remains 20

If 360 Men be in Garifon, and have Provision for 6 Months, but knowing of no additional Provision for 9 Months, how many Men must be turned out, that the Provision may last the whole 9 Months.

The Question is thus - - - 6 : 360 :: 9 : 240.

Here the Answer is 240 Men to be kept in Garifon, and the rest of the 360, to wit 120 Men, are to be put out.

9) 210 (240 Men
18
36
36
00

Note

Nota. It will often occur in Questions of this Rule, that there must be some previous Preparation, before you can state the Question for Operation, as will appear in the following Examples.

A Merchant buys 64 Tuns of Wine for 460 *lib.* Sterling, on which he pays the following Charges, viz. 220 *lib.* for Freight. 10 *lib.* for Loading and Unloading. 15 *lib.* for Custom, and 8 *lib.* of Cellar Rent. He designs to sell the Wine at such Rate as to gain 250 *lib.* on the Bargain, a Gentleman offers to buy 24 Tuns of the Wine; The Question is, what he must get for the 24 Tuns.

The previous Preparation is to sum up the whole Charge and Cost of the Wine, adding thereto the Profit desired. Thus,

	<i>lib.</i>
First Cost - - -	460
Freight - - -	220
Loading, &c. - -	10
Custom - - -	15
Cellar Rent - - -	8
Profit - - -	250
Total - - -	963

Then say

$$64 : 963 :: 24 :$$

$$\begin{array}{r} 24 \\ \hline 3852 \end{array}$$

$$1926 \text{ lib. } \text{sh. } d.$$

$$64) 23112 \quad (361 : 2 : 6$$

$$\begin{array}{r} 192 \\ \hline 391 \end{array} \text{ The Answer}$$

391 for the Price

384 of 24 Tuns.

$$\begin{array}{r} 72 \\ \hline 64 \end{array}$$

Remains - - 8 which being
Reduced, is 2 *sh.* 6 *d.*

If 60 Gallons of Water do in 1 Hour fall into a Cistern, holding 200 Gallons, and by a Pipe in the Cistern, there runs out 45 Gallons, in one Hour, in how many Hours will the Cistern be filled,

First

Arithmetick compendix'd.

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First, Find how much more runs in, than runs out in one Hour, by subtracting the one from the other, thus,

Runs in - - 60
Runs out - - 45
Difference - 15

Then say,

15 : 1 :: 200 : 13 $\frac{1}{3}$

200

15) 200 (13 $\frac{1}{3}$ Answer.

15

which is 13 Hours
and 20 Minutes.

50

45

Remains 5

A Farmer sends his Servant to Market with 216 *lib.* to buy Cattel, viz. Oxen at 11 *lib.* Cows at 8 *lib.* and Sheep at 3 *lib.* the Piece, and to bring Home a like Number of each, how many must he buy of each Sort.

First add all the different Prices together, thus,

1 Ox - - 11
1 Cow - - 8
1 Sheep - - 3
Total - - 22

Then say

22 : 1 :: 216 : 9.

12) 216 (9 Answer.

198

Remains 18 *lib.* of the Money.

So it appears that he must buy 9 of each Sort, and bring back 18 *lib.* of the Money.

Two Persons, *A.* and *B.* are to go one Journey; *A.* goes off 3 Days before *B.* and travels 30 Miles a Day; *B.* follows, and travels 50 Miles a Day: In how many Days, and after how many Miles travel will *B.* overtake *A.*

First, find how many Miles *B.* exceeds *A.* in one Day. Next find how many Miles *A.* has travelled before *B.* began his Journey, then state your Question as follows.

B. travels 50

A. travels 30

Difference 20

30 Miles *A.*'s daily Travel,

3 Days before *B.*

90 Miles.

H

Then

Then say,

$$20 : 1 :: 90 : 4\frac{1}{2}$$

90

$$20 \overline{) 90} \left(4\frac{1}{2} \text{ or } \frac{1}{2} \text{ Days. Answer}$$

80

10

So it appears that *B.* will overtake *A.* in 4 Days and a Half. And travelling 50 Miles each Day makes 225 Miles.

Proof of the Rule of Three.

If the Question be of the Direct Rule. Then multiply the 4th Number (or Answer of the Question) by the first Number, and if the Product be equal to the Product of the 2d and 3d Numbers multiplied together, your Work is right.

If your Question be of the Indirect Rule. Then multiply the 4th Number (or Answer of the Question) by the 3d Number, and if the Product be equal to the Product of the first and second Numbers multiplied together, your Work is right.

But in Case these do not hold, you must do the Work over again to discover your Error.

Take the first two Examples thus,

Direct Rule.

$$4 : 19 :: 8 : 38.$$

8

$$4 \overline{) 152} \left(38 \text{ Answer.}$$

12

4

$$\underline{32} \quad 152 \text{ Proof.}$$

32

0

Indirect Rule.

$$8 : 12 :: 16 : 6.$$

8

$$16 \overline{) 96} \left(6 \text{ Answer.}$$

96

16

0

36

6

96 Proof.

C H A P.

CHAP. XI.

The compound Rule of Three.

IT is also called the double Rule of Three; or, The Rule of Three repeated. In each Question there are five Numbers given to find out a sixth proportional thereto; which is commonly performed by two single Operations of the Rule of Three, as the easiest and plainest Way. Yet it may often be done another Way, as shall be shown afterwards, to satisfy the curious Learner.

In this Rule, as is said, there are five Numbers given, to find out a sixth proportional. Of these five, the first three contain a Supposition, and in the other two a Question is propounded. Thus,

If the Carriage of 20 Packs for 136 Miles cost 16 *lib.* what will the Carriage of 12 Packs for 28 Miles cost.

These 5 Numbers are thus ranked,

Packs	Miles	lib.	Packs	Miles	lib.
20	:	136	:	16	:
			:	12	:
			:	28	:
			:		:

In which observe that the first and fourth are of one Denomination. Also the second and fifth are of another Denomination. And the sixth must be of the Denomination of the third.

When you are to make your Operation, your Numbers must be thus ranked,

Packs	lib.	Packs
20	:	16
	:	12
Miles		Miles
136	:	28

In which observe, That the Term (or Number) which is of the same Denomination with the Term sought, is in the Middle; The other two Terms of the

Supposition, are set in the first Place, the one above the other ; and the two Terms upon which the Question is made, are in the third Place, the one above the other also ; In such Manner, that the uppermost in the first and third Places are of one Denomination : And the two lower Numbers also of one Denomination.

Those Rules are diligently to be observed in all Questions of the Double Rule of Three.

Now to work the foresaid Question,
Say first, $20 : 16 :: 12$

$$\begin{array}{r}
 12 \\
 \hline
 32 \\
 16 \\
 \hline
 20 \) \ 192 \ (\ 9 \text{ lib. } 12 \text{ sh. Answer.} \\
 \underline{180} \\
 12 \\
 20 \\
 \hline
 2 \ 10 \) \ 24 \ 10 \\
 \underline{12} \text{ Shill.}
 \end{array}$$

Here the Answer in the first Operation is 9 lib. 12 sh.

Then say, $136 : 9 : 12 :: 28$

$$\begin{array}{r}
 20 \\
 \hline
 192 \text{ Shill.} \\
 28 \\
 \hline
 1536 \\
 384 \\
 \hline
 136 \) \ 5376 \text{ sh.} \ (\ 39 \frac{72}{136} \text{ Shill. which} \\
 \underline{408} \text{ being reduced, makes} \\
 1290 \text{ 1 lib. 19 sh. 6 d. and } \frac{6}{17} \\
 \underline{1224} \text{ as the Answer.} \\
 72
 \end{array}$$

So if 20 Packs carried 136 Miles, cost 16 lib. 12 Packs, carried 28 Miles, will cost 1 lib. 19 sh. 6 $\frac{6}{17}$ d.

Example

Example 2. If 100 *lib.* in 12 Months, give 6 *lib.* of Annualrent, how much Annualrent will 500 *lib.* give in 8 Months, at the same Rate,

The State is thus,

$$\begin{array}{rcl} 100 & : & 6 :: 500. \\ 12 & & 8 \end{array}$$

The first Operation is

$$\begin{array}{r} 100 : 6 :: 500. \\ \quad 500 \\ 100 \overline{) 3000} \quad (30 \text{ lib.} \end{array}$$

The second Operation,

$$\begin{array}{r} 12 : 30 :: 8 \\ \quad 8 \\ 12 \overline{) 240} \quad (20 \text{ lib. the Answer.} \\ \quad 24 \\ \quad \underline{\quad} \\ \quad 00 \end{array}$$

When you would work any Question in the Double Rule of Three, by one compound Operation; you must take notice, That some Questions are Direct and some Indirect, as it is in the single Rule already taught.

When the Question is to be wrought by the Compound Rule Direct, observe this

Rule. After your Numbers are ranked as formerly directed, multiply the two first Terms, the one by the other, and take their Product for the first Term; Take your Middle Number for the second Term; And the Product of the two last Terms multiplied, for the third Term; This done, work by the Rule of Three direct for a Fourth Number, which is the Answer of your Question.

Example. If 4 Students spend 19 *lib.* in 3 Months, how much will 8 Students spend in 9 Months.

See

See the Work.

Stud. lib. Stud.

Here 4 mul- 4 : 19 :: 8
tiplid by 3 Mo. Mo. Here 8 multiplied by
makes 12. $\frac{3}{12}$ $\frac{9}{72}$ 9 makes 72.

Then your Work will be as below.

$$\begin{array}{r}
 12 : 19 :: 72 \\
 \hline
 72 \\
 38 \\
 \hline
 133 \\
 12) 1368 \quad (114 \text{ lib. Answer.} \\
 \hline
 12 \\
 \hline
 16 \\
 12 \\
 \hline
 48 \\
 48 \\
 \hline
 0
 \end{array}$$

When a compound Question is propounded, if any of the single Questions are to be resolved by the Indirect Rule of Three, Then observe the following Directions.

Having ranked your Numbers as already directed: Multiply the higher Number in the third Place by the lower Number in the first Place, and take that Product for your first Term, take the Middle Number for your second Term, then multiply the higher Number in the first Place by the lower Number in the third Place, and that Product is your third Term. Then consider whether the Indirect Proportion lies in the upper Line (or Question) or in the lower: If it lie in the upper, then work the compound Operation by the Rule of Three

Arithmetick compendiz'd.

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Three Direct: But if the indirect Proportion lie in the lower Line, then work by the Indirect Rule.

Example. If 9 Pecks of Corn serve 8 Horses 12 Days, how long will 24 Pecks serve 16 Horses.

See the Work,

	Horse	Days	Horse	
Here 16 multi-	8	:	12	:
plied by 9	Pecks		Pecks	Here 8 multi-
makes 144.	9	:	24	plied by 24
	<u>144</u>		<u>192</u>	makes 192.

And the Indirect Proportion lying in the upper Line, your compound Operation is as follows, by the Direct Rule.

144	:	12	:	:	192	
					12	
					<u>384</u>	
					192	
144	)	2304		(16 Days		
		144		the Answer,		
		<u>864</u>				
		864				
		<u>0</u>				

But in Case the foresaid Question be stated thus,

Pecks	Days	Pecks
9	:	12
Horse	:	24
8	:	Horse.
	:	16

Then the indirect Proportion will be found in the lower Line, and the Operation will be as follows, by the Indirect Rule, for the Question runs thus; If 9 Pecks of Corn are eaten in 12 Days by 8 Horses, in how many Days will 24 Pecks be eaten by 16 Horses.

See

See the Work.

$$\begin{array}{r}
 9 : 12 :: 24 \\
 8 : \quad 16 \\
 \hline
 32 \\
 16 \\
 \hline
 192 \\
 12 \\
 \hline
 384 \\
 192 \\
 \hline
 144) 2304 \quad (16 \text{ Days, Answer, which is the} \\
 144 \\
 \hline
 864 \\
 864 \\
 \hline
 \end{array}$$

same as in the other Operation.

I have made all the Questions and Operations in the Rule of Three, whether Single or Compound, as plain and easy as possible; By which the attentive Learner may attain to a distinct Notion of this excellent Rule, and understand how to work any Question, that may be propounded; when he has duly considered the Nature thereof, and how the Numbers are to be ranked, and what previous Reductions or other Preparations are to be made according to the preceeding Rules.

CHAP. XII.

Of Fractions.

A Fraction is one or more Parts of an Integer or Unit, according as the same is divided.

A Fraction is either *Proper* or *Improper*, *Single* or *Compound*.

A Fraction hath two Parts, viz. A *Numerator* and a *Denominator* writ thus $\frac{3}{4}$ Numerator
Denominator. The Figure below the Line or Denominator shews how many Parts the Integer is divided into; the Numerator or Figure above the Line denotes how many of these Parts are contain'd in the Fraction, so $\frac{3}{4}$ is three Parts of four, or three fourth Parts of an Integer.

A *proper Fraction* is, when the Numerator is less than the Denominator, as $\frac{1}{2}$ $\frac{3}{4}$ $\frac{7}{8}$, &c.

An *improper Fraction* is, when the Numerator is greater than the Denominator, or equal to it, so such Fractions are either greater than, or equal to, an Unit. Thus $\frac{8}{4}$ of a Pound is 4 lib. also $\frac{4}{4}$ of a Pound is 1 lib. $\frac{7}{4}$ of a Pound is 4 lib. and $\frac{1}{4}$.

A *Single Fraction* is, when there is but one Numerator, and one Denominator, as $\frac{3}{4}$ or $\frac{2}{1}$, &c.

A *Compound Fraction*, is a Fraction of a Fraction, or a Part or Parts of a Fraction, and hath several Numerators and Denominators, with the Word of interveening, as $\frac{1}{2}$ of $\frac{1}{2}$, or $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$.

In many Arithmetical Operations, there arises a *Mixt Number*, which consists of an Unit or Units, with a Fraction adjoin'd, as $2\frac{1}{2}$, or $4\frac{3}{4}$, &c.

In the Doctrine of Fractions, the Rules of Reduction are first to be exactly Taught; therefore let the Learner study the Rules following.

1. How to find the greatest common Measure to any two Numbers, when they can be equally divided
I without

without leaving a Remainder in either. This is useful for the bringing great Fractions to their lowest Terms, for making an Operation easie.

Rule. Divide the Denominator by the Numerator, then divide the Divisor by the Remainder, and so go on dividing the Divisor by the Remainder, until there be no Remainder, and the last Divisor will be a common Divisor to both the Numbers given.

Example. To find the greatest common Divisor to this Fraction $\frac{555}{629}$.

See the Work.

Here, first divide 629 by 555, there remains 74, by which divide 555, there remains 37; by which divide 74, there remains nothing, so 37 is the greatest common Divisor. Then

divide 555 by 37, the Quotient will be 15, for a new Numerator; also divide 629 by 37, the Quotient will be 17, for a new Denominator. So $\frac{15}{17}$ is equal to $\frac{555}{629}$ and in much lower Terms.

Or if you make your Division upon the Denominator continually till you find no Remainder, the Issue will be the same; so you may choose either of the Ways. See the former Example again repeated.

In this Operation the last Divisor is 37, as in the former: So any of the Ways holds good; and the Learner may choose either.

$$\begin{array}{r}
 555 \overline{) 629} (1 \\
 \underline{555} \\
 74 \overline{) 555} (7 \\
 \underline{518} \\
 37 \overline{) 74} (2 \\
 \underline{74} \\
 0
 \end{array}$$

$$\begin{array}{r}
 555 \overline{) 629} (1 \\
 \underline{555} \\
 74 \overline{) 629} (8 \\
 \underline{592} \\
 37 \overline{) 629} (17 \\
 \underline{37} \\
 259 \\
 \underline{259} \\
 0
 \end{array}$$

But

But when a Fraction consists of even Numbers, the Work of Reduction may be easier by halving, as in this Example, $\frac{32}{128}$ may be reduced to $\frac{1}{4}$, after this Manner, $\frac{32}{128}$ one Half is $\frac{16}{64}$, one Half is $\frac{8}{32}$, one Half is $\frac{4}{16}$, one Half is $\frac{2}{8}$, one Half is $\frac{1}{4}$.

By this Rule, the attentive Learner may know how to reduce a Fraction by taking $\frac{1}{2}$, or $\frac{1}{4}$, or $\frac{1}{8}$, &c. of the Numerator and Denominator, as in this Example, $\frac{28}{84}$ one Quarter is $\frac{7}{21}$, one Seventh is $\frac{1}{3}$.

When a Fraction ends with 5, or a Cypher, both in Numerator and Denominator, or with 5 in the one, and 0 in the other, such Fractions may be reduced, by taking one Fifth, or one Tenth of both, as $\frac{25}{75}$, one Fifth whereof is $\frac{5}{15}$; and so $\frac{15}{45}$, one Tenth whereof is $\frac{3}{9}$, which again may be further reduced by taking one Third of each, which brings it to $\frac{1}{3}$.

2. To reduce an *improper Fraction*, to its Equivalent, Whole or Mixt Number.

Rule. Divide the Numerator by the Denominator, and the Quotient is the integral Parts.

So $\frac{8}{2}$ is 4 Integers; for 8 divided by 2. the Quotient is 4.

But if there be a Remainder after the Division, that Remainder is a Numerator to the first Denominator, to be join'd to the Integers, which makes the Quotient a Mixt Number. So $\frac{35}{4}$ will be $8\frac{3}{4}$, for 35 divided by 4, the Quotient is 8, and 3 remaining, which set over 4 makes $\frac{3}{4}$.

3. To reduce a *whole Number* into an *improper Fraction*.

Rule. Set the whole Number as the Numerator, and 1 or an Unit as the Denominator; so 8, will be $\frac{8}{1}$, or 14 will be $\frac{14}{1}$.

4. To reduce a *Mixt Number* to an *improper Fraction*.

Rule. Multiply the Integers by the Denominator of the Fraction, and to the Product add the Numerator; and that total Product is the Numerator to the Denominator. Thus $8\frac{3}{4}$ will be $\frac{63}{4}$; For 4 Times 8 is 32,

to which add the Numerator 3, makes the new Numerator 35.

5. To reduce *Fractions of several Denominations* into *Fractions of like Value*, but having one common Denominator.

Rule. Multiply the Numerator of each Fraction separately into all the Denominators except its own, and the Product of each is a new Numerator: Then multiply all the Denominators into one another, and the Product thereof is the common Denominator to all the new Numerators.

Example. Reduce $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{7}$ to Fractions of one Denomination.

See the Work.

2. the 1st Numerator.	Then for the Denominator.
4. the 2d Denominator.	3. the 1st Denominat.
8. Product.	4. the 2d Denominat.
7. the 3d Denominator.	12. Product.
Total 56. the 1st new Numer.	7. the 3d Denominat.
3. the 2d Numerator.	84 Total new Denomin-
2. the 1st Denominator.	
9. Product.	
7. the 3d Denominator.	So the new Fractions
Total 63. the 2d new Numer.	will be these,
5. the 3d Numerator.	
4. the 2d Denominator.	$\frac{56}{84}$ } equal to $\frac{2}{3}$
20. Product.	$\frac{63}{84}$ }
3. the 1st Denominator.	$\frac{60}{84}$ }
Total 60. the 3d new Numer.	

6. To reduce a *compound Fraction* to a single Fraction of the same Value.

Rule. Multiply all the Numerators into one another, and take the Product for a new Numerator: Also multiply all the Denominators into one another, and take the Product for a new Denominator.

Example

Example. Reduce $\frac{1}{8}$ of $\frac{2}{3}$ of $\frac{7}{8}$ into a simple Fraction.

See the Work.

1. the 1 st Num.	8. the 1 st Deno.	So the Answer is $\frac{1}{192}$, which is equal to $\frac{1}{8}$ of $\frac{2}{3}$ of $\frac{7}{8}$
2 the 2 ^d Num.	3. the 2 ^d Deno.	
2. Product.	24. Product.	
7. the 3 ^d Num.	8. 3 ^d Denomin.	
14 Total Num.	192. Total Deno.	

7. To reduce a Fraction of a lower Denomination, to a Fraction of a higher Denomination.

Rule. Consider how many Units of the Denomination of your Fraction, make an Unit of the next higher Denomination, and next how many of that make one Unit of the next superior Denomination, and so on till you come to the highest you design, and set all down in the Form of a compound Fraction, and then work as in the last Case.

Example. Reduce $\frac{2}{3}$ of a Penny into the Fraction of a Pound. Here, consider, That 12 *d.* make one Shilling, so $\frac{2}{3}$ of a Penny is $\frac{2}{3}$ of $\frac{1}{12}$ of a Shilling; next consider, that 20 *sh.* make 1 *lib.* therefore $\frac{2}{3}$ of a Penny is $\frac{2}{3}$ of $\frac{1}{20}$ of a Pound. And by the Rule in the 6th Case this compound Fraction will be reduced to $\frac{2}{720}$ of a Pound, which is equal to $\frac{2}{3}$ of a Penny.

8. To reduce a Fraction of a high Denomination, to a Fraction of a lower Denomination.

Rule. Multiply the Numerator by the Number of Units in the next inferior Denomination as make an Unit of the Denomination of your Fraction, and multiply that Product by the Number of Units in the next lower Denomination, as make an Unit of the last Denomination, and so on till you come to the lowest; and take the last Product as a Numerator to your first Denominator.

Example. Reduce $\frac{3}{294}$ of a Pound to the Fraction of a Penny.

See

See the Work.

$$\begin{array}{r}
 3 \\
 20 \\
 \hline
 60 \\
 12 \\
 \hline
 120 \text{ Shill.} \\
 6 \\
 \hline
 720 \text{ Pence.}
 \end{array}$$

So the Fraction is $\frac{710}{294}$ of a Penny, which is equal to $2\frac{1}{2}$ of a Pound.

9. To discover the Value of any vulgar Fraction in the known Parts of the Integer.

Rule. Multiply the Numerator of your Fraction, by the Number of Units of the next inferior Denomination, that make an Unit of the Denomination of the Fraction, and divide that Product by the Denominator, and the Quotient is the Answer. And if there be any Remainder, you may find the Value thereof, by multiplying it by the Number of Units in the next lower Denomination, which make an Unit of the preceeding Denomination, and dividing the Product by the first Denominator, and so on till you come to the lowest Denomination you can.

Example. What is the Value of $\frac{133}{48}$ of a Pound.

See the Work.

133 the Numerator
by - - - - 20 Shill. in a Pound.

Denominat. 48|0) 266|0 (5 Shillings.

240

260 remains

by - - - - 12 Pence in a Shilling.

52

26

48|0) 312|0 (6 Pence

288

24 remains, which is $\frac{24}{48}$ of a

Penny, which reduced to its lowest Terms is $\frac{1}{2} d$. So the Value of $\frac{133}{48}$ of a Pound, is 5 lb . 6 $\frac{1}{2} d$.

C H A P. XIII.

Addition of vulgar Fractions.

THE general Rule for Addition is, to reduce all the Fractions given into one Denomination, by the Fifth Rule of Reduction; This being done, add all the new Numerators together, and take the Total for the Numerator, to the new common Denominator; Which is the Total of all the Fractions.

Example first, $\frac{1}{18}$ $\frac{3}{18}$ $\frac{5}{18}$ and $\frac{6}{18}$ to be added together will amount to $\frac{15}{18}$. For 1, 3, 5 and 6 being added, make 15, all the Denominators being already alike.

Example second, Add $\frac{2}{5}$ $\frac{3}{7}$ and $\frac{1}{8}$ together.

First, Reduce them to a common Denomination, so they will be $\frac{112}{324}$ $\frac{120}{324}$ and $\frac{35}{324}$. Then adding all the Numerators together, they amount to 267, so the Total is $\frac{267}{324}$.

To add Mixt Numbers together.

Rule. First, Add the Fractions according to the Rule above, then add the Integers joining thereto the Amount of the Fractions, so you have the compleat Total.

Example. $4\frac{5}{8}$ and $5\frac{7}{8}$ to be added.

First, Add $\frac{5}{8}$ and $\frac{7}{8}$ they make $1\frac{12}{8}$, which added to 4 and 5, makes 10 $\frac{12}{8}$ or 10 $\frac{3}{2}$.

To add compound Fractions.

Rule. Reduce your compound Fractions to simple Fractions, then add these together according to the preceeding Rules.

Example. $\frac{1}{2}$ of $\frac{1}{2}$ and $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ to be added together.

First,

First, Your compound Fractions being reduced to simple Fractions, will be $\frac{1}{2}$ and $\frac{4}{14}$; which being brought to one Denomination, will be $\frac{24}{144}$ $\frac{24}{144}$, then adding the Numerators together, makes the Total $\frac{48}{144}$ or $\frac{1}{3}$.

C H A P.

C H A P. XIV.

Subtraction in vulgar Fractions.

AS in Addition the Fractions given were reduced to one Denomination, so must they be in Subtraction, and that Reduction being made, subtract the Numerator of the lesser Fraction from the Numerator of the greater, and the Remainder is the Answer.

Example. Subtract $\frac{3}{4}$ from $\frac{5}{4}$, remains $\frac{2}{4}$.

In Subtraction there are three Cases wherein Reduction must be used before we proceed to the Work of Subtraction:

1. When the Fractions given are of different Denominations; they must be reduced to one Denomination.

Example. Subtract $\frac{2}{3}$ from $\frac{7}{6}$. Having reduced these to one Denomination, they will be $\frac{1}{2}$ and $\frac{2}{3}$. Then subtracting the lesser Numerator from the greater, the Remainder is the Answer. So here 18 subtracted from 21, remains 3, which is $\frac{1}{6}$ or $\frac{1}{2}$.

But when the Fractions given are of one Denomination, the Work is done by simple Subtraction of the lesser Numerator from the greater, as above.

Example. Subtract $\frac{2}{3}$ from $\frac{4}{3}$, remains $\frac{2}{3}$.

2. When one of the Fractions is a compound Fraction: The Compound must be reduced to a simple Fraction.

Example. Subtract $\frac{1}{2}$ of $\frac{1}{3}$ from $\frac{7}{8}$; Here the compound Fraction being reduced is $\frac{1}{6}$. Then reducing this $\frac{1}{6}$, and the other Fraction $\frac{7}{8}$ into one Denomination, will be $\frac{8}{48}$ and $\frac{4}{48}$, lastly subtracting the lesser Numerator from the greater, the Remainder will be $\frac{4}{48}$ or $\frac{1}{12}$.

3. When whole or mixt Numbers occur, as a Fraction to be subtracted from a whole or mixt Number; Or a mixt Number to be subtracted from a mixt Number; or lastly, a whole Number to be subtracted from a mixt Number.

The general Rule is, Reduce the whole or mixt Numbers to improper Fractions; then reduce these to one Denomination, and lastly, subtract the lesser from the greater, so you have the Answer.

Example. What is the Difference betwixt $4\frac{1}{3}$ and $2\frac{2}{3}$. These reduced to improper Fractions are $\frac{13}{3}$ and $\frac{8}{3}$. Which being of one Denomination, the Difference is $\frac{5}{3}$ or $1\frac{2}{3}$.

Another Example. What is the Difference betwixt 8 and $3\frac{1}{4}$. These reduced will be $\frac{8}{1}$ and $\frac{13}{4}$, which brought to one Denomination will be $\frac{32}{4}$ $\frac{13}{4}$; And the lesser subtracted from the greater there remains $\frac{19}{4}$ or $4\frac{3}{4}$.

Example. 3. What is the Difference betwixt $7\frac{1}{2}$ and 4. These reduced will be $\frac{15}{2}$ and $\frac{4}{1}$, which brought to one Denomination are $\frac{15}{2}$ $\frac{8}{2}$, and the lesser subtracted from the greater, there remains $\frac{7}{2}$ or $3\frac{1}{2}$.

But the Work of Subtraction when mixt or whole Numbers occur may be oft contracted, as in the Examples following.

Subtract $3\frac{1}{2}$ from 9. Here subtract $\frac{1}{2}$ from 1 Integer there remains $\frac{1}{2}$, then subtract 3 out of 8 (the first 9 being lessen'd by 1) there remains 5, so the whole Remains join'd is $5\frac{1}{2}$.

Subtract 4 from $7\frac{1}{4}$. Here subtract 4 from 7, there remains 3, to which add the Fraction join'd to 7, makes $3\frac{1}{4}$ for the Remainder.

Subtract $9\frac{1}{4}$ from $15\frac{3}{4}$. Here subtract $\frac{1}{4}$ from $\frac{3}{4}$, there remains $\frac{2}{4}$ or $\frac{1}{2}$, then subtract 9 from 15, there remains 6. So the two Remainders being join'd make $6\frac{1}{2}$ in all.

Subtract $14\frac{2}{3}$ from $19\frac{1}{3}$. Here, because the Fraction of the lesser Number, is greater than the Fraction of the greater Number, therefore you must reduce an Unit of the greater Number, with its Fraction, into an improper Fraction, and then the Numbers will stand thus, $18\frac{4}{3}$ and $14\frac{2}{3}$ then subtract, as in the last Example, and the Remainder will be $4\frac{2}{3}$, thus, $\frac{2}{3}$ from $\frac{4}{3}$, remains $\frac{2}{3}$, and 14 from 18, there remains 4.

C H A P. XV.

Multiplication in vulgar Fractions.

IN this Rule there is no need to reduce simple Fractions to one Denomination, as in the two preceeding Rules. *But* if a compound Fraction be given, it must be reduc'd to a simple; *Also* if mixt Numbers or whole Numbers be given, these must be reduc'd to improper Fractions; Then work by this

Rule. Multiply the Numerators into one another, and take the Product for a new Numerator; Also multiply the Denominators into one another, and take the Product for a new Denominator; which two make the Product sought.

Example, 1st. Multiply $\frac{2}{3}$ by $\frac{3}{5}$. They make $\frac{6}{15}$, or $\frac{2}{5}$.

Example 2^d. Multiply $\frac{4}{7}$ by $\frac{1}{4}$ of $\frac{2}{3}$. Here the compound Fraction being reduced makes $\frac{2}{12}$, which multiplied by $\frac{4}{7}$, makes $\frac{8}{84}$ or $\frac{2}{21}$.

Example 3^d. Multiply 7 by $\frac{3}{5}$. Here the whole Number being brought to an improper Fraction is $\frac{7}{1}$, which multiplied by $\frac{3}{5}$ makes $2\frac{1}{5}$, or $4\frac{1}{5}$.

Example 4. Multiply $23\frac{3}{4}$ by $19\frac{5}{8}$. Here both Numbers being reduc'd to improper Fractions, will be $9\frac{3}{4}$ and $11\frac{5}{8}$, which multiplied together, will be $113\frac{15}{32}$, which is $47\frac{1}{2}$.

CHAP

CHAP. XVI.

Division in vulgar Fractions.

IN this Rule, as in Multiplication, there is no need to reduce simple Fractions to one Denomination. But if a compound Fraction be given, it must be reduc'd to a simple Fraction. Also if mixt Numbers or a whole Number be given, these must be reduc'd to improper Fractions ; Then work by this

Rule, Multiply the Numerator of the Dividend, by the Denominator of the Divisor, and take the Product for the Numerator of the Quotient. Also multiply the Denominator of the Dividend by the Numerator of the Divisor, and the Product is the Denominator of the Quotient.

Example 1st. Divide $\frac{1}{2}$ by $\frac{1}{2}$. Place your Divisor and Dividend as in whole Numbers ; Then work by the Direction given. Thus,

$$\frac{1}{2}) \frac{1}{2} (\frac{2}{2} \text{ Answer. Which is } 1 \frac{0}{2}.$$

Example 2^d. Divide $\frac{7}{8}$ by $\frac{1}{6}$ of $\frac{2}{5}$. Here the compound Fraction being reduc'd is $\frac{2}{30}$ or $\frac{1}{15}$; Then the Work stands thus, $\frac{1}{15}) \frac{7}{8} (\frac{105}{8} \text{ Answer, which is } 13 \frac{1}{8}.$

Example 3^d. Divide $5 \frac{3}{7}$ by $2 \frac{3}{5}$. Here both Numbers being reduc'd to improper Fractions, will be $\frac{38}{7}$ by $\frac{13}{5}$, Then the Work stands thus,

$$1 \frac{3}{5}) \frac{38}{7} (\frac{190}{7}, \text{ which is } 2 \frac{6}{7}.$$

Example 4. Divide 7 by $\frac{2}{5}$. Here 7 must be brought to an improper Fraction. Then the Work stands thus,

$$\frac{2}{5}) 7 (\frac{35}{2}, \text{ which is } 17 \frac{1}{2}.$$

Having

Having now explain'd the Doctrine of Vulgar of all its Parts; And the Golden Rule, both Single and Compound, being fully treated of in whole Numbers, it is needless to repeat that Rule in Fractions, the same being alike both in whole Numbers and Fractions as to Method; Only when a Question of the Rule of Three falls to be in Fractions, the preceeding Rules, for Fractional Operations, are to be observed.

C H A P.

C H A P. XVII.

Decimal Arithmetick.

A Decimal Fraction always supposes the Integer to be divided into 10, or 100, or 1000, or 10000 Parts, &c. to what Degree you please for Exactness: Hence the Denominator is always known, and needs not be writ; but you place and use your Fraction as an Integer, taking Care only to prefix the *Decimal Point*, for distinguishing it from an Integer, so $\frac{5}{10}$ is writ thus .5; and $\frac{27}{100}$ is writ .27 and $\frac{149}{1000}$ is writ .149: In which observe, That as many Figures as there are in the Numerator, so many Cyphers there are in the Denominator with an Unit prefixt in Place of the Point in the Numerator.

When a Decimal Fraction has not so many Figures in the Numerator as there are Cyphers in the Denominator, the void Places are filled up with Cyphers on the left Hand, so $\frac{5}{100}$ is writ thus .05 also $\frac{3}{1000}$ is writ thus .003, &c. Whence it appears, that as in whole Numbers Cyphers on the right Hand, *increase* the Number or Value, so in Decimals, Cyphers on the left Hand, *diminish* the Value, as in the following Table.

Integers	$\left\{ \begin{array}{l} 5 \\ 50 \\ 500 \\ 5000 \end{array} \right\}$	is	$\left\{ \begin{array}{l} \text{Five} \\ \text{Fifty} \\ \text{Five hundred} \\ \text{Five thousand.} \end{array} \right.$
Decimals	$\left\{ \begin{array}{l} .5 \\ .05 \\ .005 \\ .0005 \end{array} \right\}$	is	$\left\{ \begin{array}{l} \frac{5}{10} \text{ Five, tenth Parts} \\ \frac{5}{100} \text{ Five, hundredth Parts} \\ \frac{5}{1000} \text{ Five, thousand Parts} \\ \frac{5}{10000} \text{ Five, tenthousand Parts} \end{array} \right.$

So,

So, as in whole Numbers, the Value increaseth from the *right Hand* to the *Left*. In Decimals, the Value decreaseth from the *left Hand* to the *Right*; as in the Table below.

7	6	5	4	3	2	1.	2	3	4	5	6	7	8
Integers.							Decimals.						
Millions. Hundreds of Thousands. Tens of Thousands. Thousands. Hundreds. Tens. Units.							Tenth Parts. Hundred Parts. Thousand Parts. Ten thousand Parts. Hundred thousand Parts. Million Parts. Ten Million Parts.						

Observe also, That as in whole Numbers Cyphers on the left Hand, neither increase nor diminish the Value: So in Decimals, Cyphers on the right Hand, neither increase nor diminish the Value. Thus 025 in whole Numbers is no better than 25. And in Decimals .500 is no better than .5

What a Decimal is, being now explain'd, the first Rule, for Operation, to be Taught is Reduction.

C H A P. XVIII.

Reduction in Decimals.

HERE it's proper to remark that in reducing vulgar Fractions to Decimals; The Decimal will not, alwise, be of full Value with the vulgar; But yet it may be brought so near, by increasing the Number of Places, that the Defect needs not be regarded, as will hereafter appear; and for the greatest of ordinary Practice 5 or 6 Places is sufficient; tho' in calculating Tables more Places are needful.

Now in reducing a vulgar Fraction to a Decimal, the Proportion is this; As the Denominator of the Fraction given is to its Numerator, so is 10, 100, 1000, 10000, &c. to the Decimal whose Denominator is 10, 100, 1000, or 10000, &c.

Suppose then $\frac{1}{8}$ given to be reduc'd to a Decimal; The State of the Question will be this, by the Rule of Three.

$$8 : 1 :: 1000$$

$$8 \overline{) 1000} \quad .125 \text{ Answer}$$

$$\begin{array}{r} 8 \\ \hline 20 \\ 16 \\ \hline 40 \\ 40 \\ \hline 0 \end{array}$$

So this Decimal .125 is exactly equal to the Fraction $\frac{1}{8}$.

Example

Example 2. Reduce $\frac{5}{7}$ to a Decimal.

See the Work.

Here, tho' there be 6 remaining in the Division, yet it is so small that it does not amount to the thousand Part of an Unit.

$$\begin{array}{r}
 7 : 5 :: 10000 (\\
 \underline{10000} \\
 7) 50000 (.7142, \text{Ans.} \\
 \underline{49} \\
 10 \\
 \underline{7} \\
 30 \\
 \underline{28} \\
 20 \\
 \underline{14} \\
 6 \text{ Remains.}
 \end{array}$$

You may take the Rule for Reduction plainly thus. Add a competent Number of Cyphers to the Numerator of the Fraction given, then divide by the Denominator, and the Quotient is the Answer. But take Care to place the Decimal Point before the Cyphers which you add, because so many Cyphers as are added in the Work, so many Places must there be in the Quotient or Answer; And if so many do not arise (as often there will not) you must supply the Defect by prefixing Cyphers on the left Hand, as in this Example 3d.

Reduce $\frac{3}{35}$ to a Decimal.

See the Work.

$$35) 3.0000 (.0857 \text{ Answer.}$$

$$\begin{array}{r}
 280 \\
 \underline{200} \\
 175 \\
 \underline{250} \\
 245 \\
 \underline{} \\
 5
 \end{array}$$

Here after finishing the Division there are but three Figures in the Quotient, and there being four Cyphers added in the Work, the Defect is made up by prefixing a Cypher, which makes the Quotient .0857.

Example

Example 4th. Reduce 14 *sh.* to the Decimal of a Pound. Here, consider that 14 *sh.* is $\frac{14}{20}$ of a Pound, So the Operation is thus,

$$\begin{array}{r} 20 \overline{) 14.0} \quad (.7 \text{ Answer.} \\ \underline{140} \\ 0 \end{array}$$

Example 5th. Reduce 13 *sh.* to a Decimal. The Work is thus,

$$\begin{array}{r} 20 \overline{) 13.00} \quad (.65 \text{ Answer.} \\ \underline{120} \\ 100 \\ \underline{100} \\ 0 \end{array}$$

Example 6th. Reduce 3 *d.* to the Decimal of a Pound.

Here you must consider, that there being 240 Pence in a Pound, so 3 Pence is $\frac{3}{240}$ of a Pound; and therefore the Work is as follows.

$$\begin{array}{r} 24 \overline{) 3.0000} \quad (.0125 \text{ Answer.} \\ \underline{24} \\ 60 \\ \underline{48} \\ 120 \\ \underline{120} \\ 0 \end{array}$$

L z

Observe

Observe that when a compound Fraction is given to be reduc'd into a Decimal, it must first be reduc'd to a simple Fraction, and that simple must be reduc'd to a decimal Fraction.

Example 7th. Reduce $\frac{1}{3}$ of $\frac{3}{4}$ to a Decimal, First, this being reduc'd to a Simple is $\frac{1}{4}$, which reduc'd to a Decimal is .25. The Learner may make the Operation.

To find the Value of a decimal Fraction, observe this

Rule, Multiply the Decimal given by such a Number of Units of the next inferior Denomination as make an Unit of your decimal Denomination, and cut off as many Figures on the right Hand of the Product, as there are Places in the Decimal, and the Figures cut off on the left Hand, are so many Integers of that Inferior Denomination: And go on repeating the like Work, till you have no Remainder, or have found the Units of the lowest Denomination you desire.

Example. What is the Value of .6125 of a Pound.

See the Work.

	.6125
	20
Shill. - -	12 25 00
	12
	50
	25
	3 00

So the Answer
is 12 sh. 3 d.

Pence = 3/00

Example

Example 2. What is the Amount of .7354 of a Year.

See the Work.

	.7354
Months in a Year	- - 12
	<u>14708</u>
	7354
Months	8 8248
Weeks in a Month	- - 4
	<u>32992</u>
Days in a Week	- - - 7
	<u>20944</u>
Hours in a Day	- - 24
	<u>3776</u>
	1888
Hours	2 2656
Minutes in an Hour	- 60
	<u>152300</u>

The whole is 8 Months 3 Weeks, two Days 2 Hours 15 Minutes and very near a Minute more.

What is the Amount of .3567 of a Chalder of Victual,

	.3567
Bolls in a Chalder	- 16
	<u>21402</u>
	3567
Bolls	5 7072
	<u>4</u>
Firlots	2 8288
	<u>4</u>
Pecks	3 152
	<u>4</u>
Lippies	1 2608

The whole is 5 Bolls 2 Firlots 3 Pecks 1 Lippie, and the Fraction of a Lippie.

The Value of the Decimal of a Pound or 20 Shill. may be known by Inspection, without any Operation by this

Rule, Double the Figure standing next to the Decimal Point, and if the Figure in the second Place be 5 or more, add one to the former Product, which together is so many Shillings; Then the Figure remaining in the second Place, and the Figure in the third Place make so many Fourth Parts of a Penny, therefore taking a Fourth Part thereof you have the Pence of the

the Decimal: With this Caution, That if the Number amount to 13 and within 39, you must deduct 1, and if it be 39 or above, you must deduct 2 before you take your Pence.

Example, .75 is 15 *sh.* for 2 Times 7 is 14, to which adding 1 for 5 standing in the second Place makes 15.

Also, .465 is 9 *sh.* 3 *d.* for twice 4 is 8, to which adding 1, because the second Figure exceeds 5, makes 9 *sh.* then there remains 15 in the second and third Places, which made less by 1, remains 14, which divided by 4 produces 3 *d.* and a Fraction remaining, which needs not be regarded, seing in *Scotland* we use no Denomination below a Penny, and there is never less than 2 *d.* paid or received.

What has been already said, is sufficient to give the ingenious Learner the right Notion of Decimals; therefore I proceed to the Rules of Addition, Subtraction, Multiplication and Division.

CHAP. XIX.

Addition of Decimals.

THERE is no Difference betwixt this and Addition of whole Numbers; But Care must be taken to place the Decimals orderly, That is, the Decimal Points all directly under one another, and the Figures of each Place in their due Order; And in the Total after Addition, you must Point off so many Figures on the right Hand, as there are Places in the greatest Decimal of the Work.

Notice also, That in Mixt Numbers; The Integers stand on the left Hand of the Decimal Point thus, 25.25, which is 25 *lib.* 5 *lb.*

Examples.

.9654	.245	22.0732	10.5763
.835	.31	12.0095	9.005
.75	.152	11.163	8.075
.006	.35	10.999	7.1463
.074	.427	6.854	6.0532
2.6304	1.484	63.0987	40.8558

Subtraction

Subtraction in Decimals.

Take Care only to set down your Numbers right as directed in Addition, and then work as in Subtraction of whole Numbers.

Examples.

From - -	39.725	11.32	.986	2.357
Subtract	18.012	9.2954	.095	1.989
Remain.	21.713	2.0246	.891	.368

C H A P.

CHAP. XX.

Multiplication in Decimals.

IN this there is no Difference from Multiplication in whole Numbers ; Except that after your Multiplication is ended, you are to observe this general

Rule, To point off, towards the right Hand, so many Figures of the Product, as there are Decimals both in the Multiplicand and Multiplier, and if so many happen not to be in the Product (as it will happen when you multiply two Fractions of little Value; or a small mixt Number by a small Fraction) you must make up the Number by annexing Cyphers on the left Hand, and then prefix the Decimal Point.

Example 1.

$$\begin{array}{r} \text{Multiply} \quad 3.467 \\ \text{by} \quad - \quad - \quad - \quad 19.01 \\ \hline 3467 \\ 31203 \\ \hline 65.90767 \\ \text{Product} \quad - \end{array}$$

Example 3.

$$\begin{array}{r} \text{Multiply} \quad .13461 \\ \text{by} \quad - \quad - \quad - \quad 42 \\ \hline 26922 \\ 53844 \\ \hline 5.65362 \\ \text{Product} \quad - \end{array}$$

Example 2.

$$\begin{array}{r} \text{Multiply} \quad 36492 \\ \text{by} \quad - \quad - \quad - \quad .032 \\ \hline 72984 \\ 109476 \\ \hline 1167.744 \\ \text{Product} \quad - \end{array}$$

Example 4.

$$\begin{array}{r} \text{Multiply} \quad .1264 \\ \text{by} \quad - \quad - \quad - \quad .247 \\ \hline 8848 \\ 5056 \\ 2528 \\ \hline .0312208 \\ \text{Product} \quad - \end{array}$$

In this 4th Example you see a Cypher added to make up the Decimals in the Product.

M

Example

Arithmetick compendiz'd.

Example 5th.

Multiply .01832
by - - - - .007

Product .00012824

In this Example three Cyphers are added to the Product, both Fractions being small, and so producing a very small Product, as is remarked in the Rule above for Multiplication.

Example 6th

Multiply 2.0051
by - - - - .0004

Product .00080204

In this Example three Cyphers are added to the Product, the Mixt Number being small and multiplied by a small Fraction.

C H A P.

C H A P. XXI.

Division in Decimals.

IN this Rule you must place your Numbers and Work as in Division of Integers : All the Difficulty being to discover the true Value of the Quotient, for which End observe this

Rule, Notice how many Decimal Places there are in the Divisor, and how many in the Dividend ; And so many as these in the Dividend exceed those in the Divisor, point off as many to the right Hand in the Quotient ; And if there be not as many in the Quotient, make up the Deficiency by prefixing Cyphers on the left Hand.

Observe also this Rule, That if in the Numbers given, there be no Decimals in the Dividend, or fewer than in the Divisor, you must add so many Cyphers on the right Hand of your Dividend, as will make the Decimals therein exceed those in the Divisor, and for ordinary 3, 4 or 5 Cyphers will be enough.

The following Examples will make all this very plain.

1 Example. Divide 742.651 amongst 41 Persons.

See the Work.

41) 742.651 (18.113, Answer.

$$\begin{array}{r}
 41 \overline{) 742.651} \\
 \underline{332} \\
 228 \\
 \underline{46} \\
 41 \\
 \underline{55} \\
 41 \\
 \underline{141} \\
 123 \\
 \underline{18}
 \end{array}$$

Remains

18

In this Quotient there are 3 Decimals pointed off equal to the Decimals in the Dividend, there being none in the Divisor. So the Answer is 18 *lib.* 2 *sh.* 3 *d.* to each, and a very inconsiderable Fraction over.

M 2

2 Example

2 Example. Divide 4672.565 by 25.635.

25.635) 4672.565000 (182.272 Answer,

$$\begin{array}{r}
 25635 \\
 \hline
 210906 \\
 205080 \\
 \hline
 58265 \\
 51270 \\
 \hline
 69950 \\
 51270 \\
 \hline
 186800 \\
 179445 \\
 \hline
 73550 \\
 51270 \\
 \hline
 22280
 \end{array}$$

Remains

In this Example, because the Decimals in both Numbers were equal, three Cyphers are added to the Dividend, and consequently three Figures are pointed off in the Quotient, which makes the Answer to be 182.272 and a small Remains, which is less than one thousand Part of an Integer.

3 Example. Divide 1425 by 6252.

6252) 1425.000000 (2279.27 Answer,

$$\begin{array}{r}
 12504 \\
 \hline
 17400 \\
 12504 \\
 \hline
 49560 \\
 43764 \\
 \hline
 57960 \\
 56268 \\
 \hline
 16920 \\
 12504 \\
 \hline
 44160 \\
 43764 \\
 \hline
 396
 \end{array}$$

In this Example, because there are four Decimals in the Divisor and none in the Dividend, therefore there are six Cyphers added to the Dividend, which is two more than in the Divisor, and consequently two Figures are cut off in the Quotient.

Nota. That all Cyphers added to the Dividend, stand only as Decimals.

4 Example

4 Example. Divide .13975, by 43.
 43) .13975 (00325, Answer.

$$\begin{array}{r}
 129 \\
 \hline
 107 \\
 86 \\
 \hline
 215 \\
 215 \\
 \hline
 0
 \end{array}$$

In this, after the Division is finished, there are but three Figures in the Quotient, and seing there ought to be five, the Deficiency is supplied by annexing two Cyphers on the left Hand.

5 Example. Divide .75 by .0125.
 .0125) .7500 (60, Answer.

$$\begin{array}{r}
 .750 \\
 \hline
 00
 \end{array}$$

In this Example observe, That by adding two Cyphers to the Dividend, the Number of Places both in

Divisor and Dividend are equal, and therefore none to be pointed off in the Quotient, so the Answer is 60 Integers.

6 Example. Divide 32.457 by 8756.323:
 8756.323) 32.45700000 (.003706

$$\begin{array}{r}
 26268969 \\
 \hline
 61880310 \\
 61294261 \\
 \hline
 58604900 \\
 52537938 \\
 \hline
 6066962
 \end{array}$$

In this sixth Example, the Number of Decimals being equal both in Divisor and Dividend, and the Divisor consisting of many Integers, there is a Necessity to add six Cyphers to the Dividend, by which the Decimals in the Dividend, exceed those in the Divisor, by six, and there arising only four Figures in the Quotient, the other two are supplied by prefixing two Cyphers.

When

When any Decimal Fraction or Mixt Number is to be divided by an Article, that is a Number consisting of an Unit and a Cypher or Cyphers, as 10, 100, 1000, &c.

Rule. Remove the Decimal Point so many Places towards the left Hand, as there are Cyphers annexed to the Unit, and when there occur blank Places fill them up with Cyphers.

Example. Suppose 17.28, given to be divided by

$$\begin{array}{rcl}
 & 10 & \\
 \text{or by} & 100 & \\
 \text{or by} & 1000 & \\
 \text{or by} & 10000 &
 \end{array}
 \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{The Quotient} \\ \text{will be} \end{array}
 \left\{ \begin{array}{l} 1.728 \\ .1728 \\ .01728 \\ .001728 \end{array} \right.$$

Hence it follows, That if the Divisor be an Article above Unity with Cyphers, as 20, 300, 4000, the Division is performed by removing the Decimal Point according to the preceeding Rule, and then taking the Half, Third, Fourth, &c. Part of the Number. Thus,

423.576 divided by 200. the Operation is thus,

$$\frac{1}{2} \text{ is } - \frac{4.23576}{2.11788} \text{ for Answer.}$$

376.4596 divided by 40, the Operation is thus,

$$\frac{1}{4} \text{ is } - \frac{37.64596}{9.41149} \text{ for Answer.}$$

5687.22 divided by 6000, the Operation is thus,

$$\frac{1}{6} \text{ is } - \frac{5.68722}{.94787} \text{ for Answer.}$$

42.33 divided by 300, the Operation is thus,

$$\frac{1}{3} \text{ is } - \frac{.4233}{.1411} \text{ Answer.}$$

I shall

I shall conclude this small Treatise with one Remark, viz. That the Issue of the Work of Multiplication and Division in Fractions, whether Vulgar or Decimal, differs from that in whole Numbers: For in whole Numbers the Product of Multiplication is always greater, both in Value and Number, than the Multiplicand; because it is the multiplying of Units, which must necessarily augment them. Whereas in Fractions the Product of Multiplication is always less in Value, tho' it may be greater in Number than the Multiplicand; because it is the Multiplication of the Parts into which an Unit is divided, which must needs make the Parts produced less in Value, tho' greater in Number.

The Reason of this is, That if a Fraction be multiplied by 1 or Unity, it produceth the Fraction given: But if a Fraction be multiplied by $\frac{1}{2}$ the Product must be the Half of the Fraction given; so $\frac{3}{4}$ multiplied by $\frac{2}{3}$, the Product must be $\frac{2}{3}$ of $\frac{3}{4}$ which is $\frac{6}{12}$ or $\frac{1}{2}$, which is less than the Multiplicand.

And Division being the Reverse of Multiplication; Therefore, as in whole Numbers the Quotient is less than the Dividend; so in Division of Fractions, the Quotient is greater than the Dividend, and often arises to Integers. All which is plainly illustrated in the preceeding Rules and Examples.

The *Golden Rule* in Decimals is the same with that in whole Numbers, observing only the Rules for decimal Operation, as before taught.

F I N I S.

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His book
Written by me
Whoever it be



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